Dark matter and energy and the quantum vacuum

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The Einstein's equation

The unknown: the metric of space-time

$$R_{\mu\nu}(x) - \frac{1}{2} g_{\mu\nu}(x) R + \Lambda g_{\mu\nu}(x) = -\kappa T_{\mu\nu}(x)$$

Space-time curvature

The cosmological constant

Energy tensor of matter

Proportionality constant
$\kappa = G / c^2$

Matter tells space-time how it must be curved; space-time tells matter how it must move

The Friedman Lemaître equations

The Einstein equation

$$\mathcal{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R} = 8\pi G_N T_{\mu\nu} + \Lambda g_{\mu\nu}$$

Matter content of the universe, perfect fluid

$$T_{\mu\nu} = -p g_{\mu\nu} + \left(p + \rho \right) u_\mu u_\nu$$

$u = (1,000)$ velocity vector for the isotropic fluid in comoving coordinates

Friedman-Lemaître equations

$$H^2 \equiv \left(\frac{\dot{R}}{R} \right)^2 = \frac{8\pi G_N \rho}{3} - \frac{k}{R^2} + \frac{\Lambda}{3}$$

Energy conservation, via $T^{\mu\nu}_{;\mu} = 0$

$$\dot{\rho} = -3H \left(\rho + p \right)$$

$$\frac{\ddot{R}}{R} = \frac{\Lambda}{3} - \frac{4\pi G}{3} \left(\rho + 3p \right)$$

Definition of cosmological parameters

$$\rho_c \equiv \frac{3H^2}{8\pi G_N}$$

$$\Omega_{\text{tot}} = \rho / \rho_c$$

$$k / R^2 = H^2 \left(\Omega_{\text{tot}} - 1 \right)$$

$$k / R_0^2 = \left(\Omega_{\text{M}} + \Omega_{\text{R}} + \Omega_{\text{V}} - 1 \right)$$

In ΛCDM

$$k = 0$$

$$\Omega_{\text{M}} = \Omega_{\text{b}} + \Omega_{\text{DM}}$$

$$\Omega_{\text{tot}} = \Omega_{\text{b}} + \Omega_{\text{R}} + \Omega_{\text{DM}} + \Omega_{\text{DE}} = 1$$

Vanishing of spatial curvature = vanishing of “total energy”?

Big bang cosmology,

K. Nakamura et al. (PDG), JP G 37 0750021 (2010)

“Eq. (19.8) has a simple classical mechanical analog if we neglect (for the moment) the cosmological term Λ. By interpreting $-k/R^2$ Newtonianly as a ‘total energy’, then we see that the evolution of the Universe is governed by a competition between the potential energy, $8\pi G_N\rho/3$, and the kinetic term $(H_0)^2$.”

However, “the usage of referring $-k/R_0^2H_0^2$ as Ω_k is unfortunate: it encourages one to think of curvature as a contribution to the energy density of the Universe, which is not correct”

Emergent perspective of gravity and dark energy

T. Padmanabhan ArXiv: 1207,0505
***Gravity and/is thermodynamics* ArXiv:1512,06546**
***Exploring the nature of gravity* ArXiv: 1602.011474**

In the conventional approach, gravity is treated as a field which couples to the energy density of matter. The addition of a cosmological constant — or equivalently, shifting of the zero level of the energy — is not a symmetry of the theory and the field equations (and their solutions) change under such a shift. In the emergent perspective, it is the *entropy density* rather than the *energy density* which plays the crucial role.

In Friedman universe in expansion, there is a horizon with radius H^{-1} to which is associated an entropy

$S = \left(A/4L_p{}^2\right) = \left(\pi / H^2 L_P{}^2\right)$ and a temperature $T = \hbar H / 2\pi$

During time interval *dt* the change of gravitational entropy is

$T\left(dS/dtn\right) = \left(1/4L_P{}^2\right)\left(dA/dt\right)$ and the corresponding heat flux

$$T\left(dS/dt\right) = \left(H/8\pi G_N\right)\left(dA/dt\right)$$

Gibbs-Duhem relation for matter

entropy density for matter is $s_m = (1/T)(\rho + P)$

with correspondig heat flux $Ts_m A = (\rho + P)A$

balancing matter and gravitational heat fluxes leads to

$TdS/dt = (\rho + P)A$ which becomes

$\frac{H}{8\pi G_N}\frac{dA}{dt} = (\rho + P)A$, which with $A = 4\pi/H^2$ gives

$$\dot{H} = -4\pi G_N(\rho + P)$$

Which is the correct Friedman equation

Energy conservation for matter for matter

$$\frac{d(\rho a^3)}{dt} = -P\frac{da^3}{dt}$$

$$\dot{\rho} = -3H(\rho + P) = \frac{3H\dot{H}}{4\pi G_N}$$

$$\rho = \frac{3H^2}{8\pi G_N} + \text{ constant} = \rho + \rho_\Lambda$$

The entropy balance condition correctly reproduces the field equation but with an arbitrary cosmological constant acting as an integration constant : the entropy density vanishes for the cosmological constant: $\rho_\Lambda = -P_\Lambda$

When the space-time responds in a manner maintaining entropy balance, it responds to

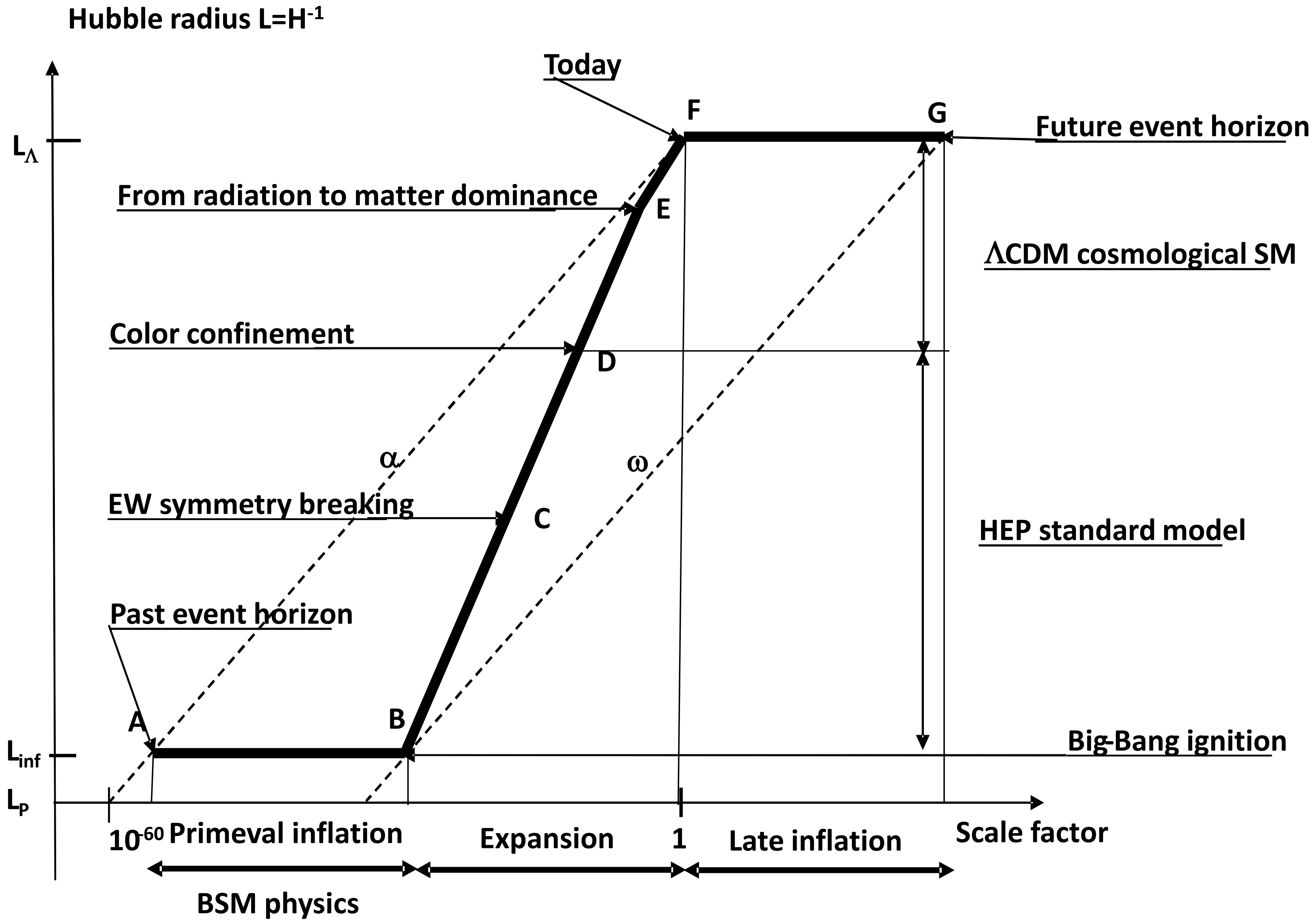
$\rho + P$ or more generally to $T_{\mu\nu} l^{\mu} l^{\nu}$ where l is a null 4-vector

which vanishes for the cosmological constant

In other words, shifting the zero level of the energy is the symmetry of the theory in the emergent perspective and gravity does not couple to the cosmological constant. The vanishing of CC would be a direct consequence of this symmetry. The smallness of CC arises as a consequence of the smallness of the symmetry breaking (**'t Hooft naturalness criterion**)

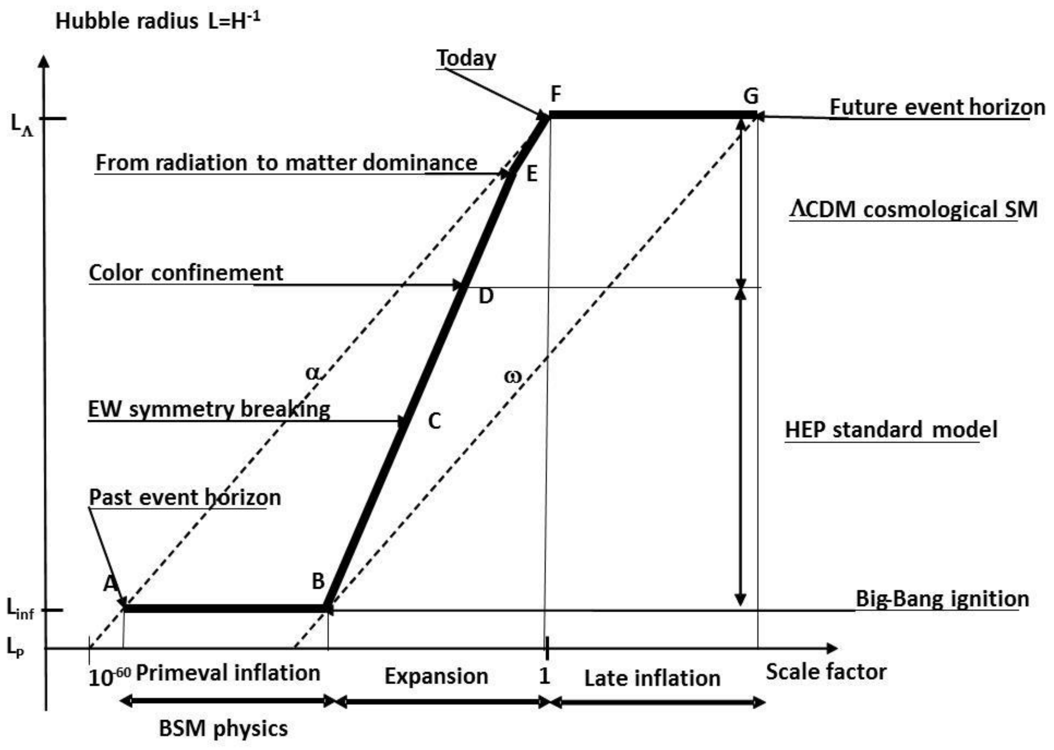
Interpreting the ΛCDM cosmology in the emergent perspective of gravity has led T. Padmanabhan to a solution of the CC problem, and as we are going to show, could lead to a new approach of the dark matter problem

The ΛCDM cosmology

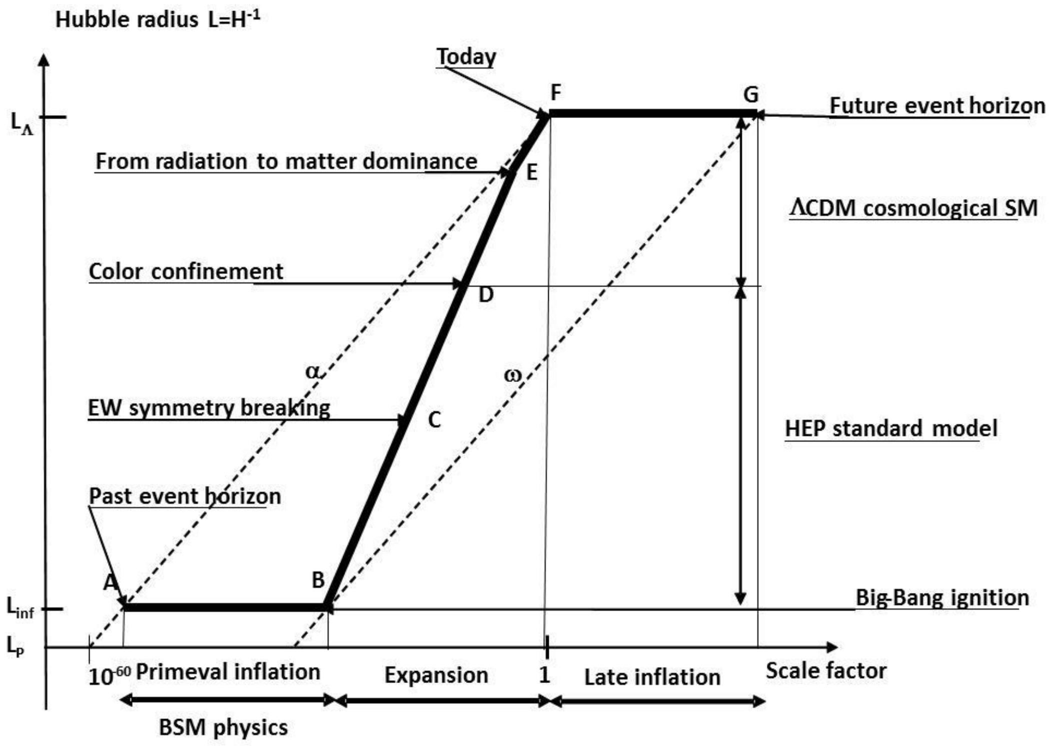


The three stages of the ΛCDM cosmology

- **The primordial inflation stage (AB)**
  - A BSM (beyond the standard model) phase (Quantum gravity? GUT?, SUSY? Superstrings?, Baryo- and lepto-genesis? Axions and axion-like particles?) that replaces the "big bang"
  - Relaxation of the **quantum vacuum** after a quantum fluctuation of matter and gravitational fields that occurred in the remote past: all the particles of the SM (supposed to be mass less before the BEH mechanism) and possible BSM particles are **virtual**
  - the scale factor grows exponentially while the Hubble radius of order of a GUT scale remains constant (de Sitter space-time geometry)
  - Inflation stops at B when the lightest non massless particles of the SM, possibly the neutrinos (with masses of about 1/1000 eV, of order of the CC scale) decohere. Point B is called in the literature the reheating point. (Bjorken calls it the ignition of the big bang).



- **The conventional FLRW phase (BF)**
  - Radiation dominated phase (BE), with L proportional to a^2 that goes through
    - the breaking of the electroweak symmetry at C by means of the BEH mechanism when all SM particles except photons and gluons became massive
    - The confinement of color at D when colorless hadrons (baryons and mesons) were formed.
  - Pressure-less matter dominated phase (EF), with L proportional to $a^{1.5}$, a phase in which occurred the events of the conventional big bang model.
- **The late inflation phase (FG)**
  - The acceleration of the expansion observed today is interpreted as due to a non vanishing CC
  - Asymptotically in the future, CC implies the existence of an event horizon at G beyond which the whole matter will be unobservable

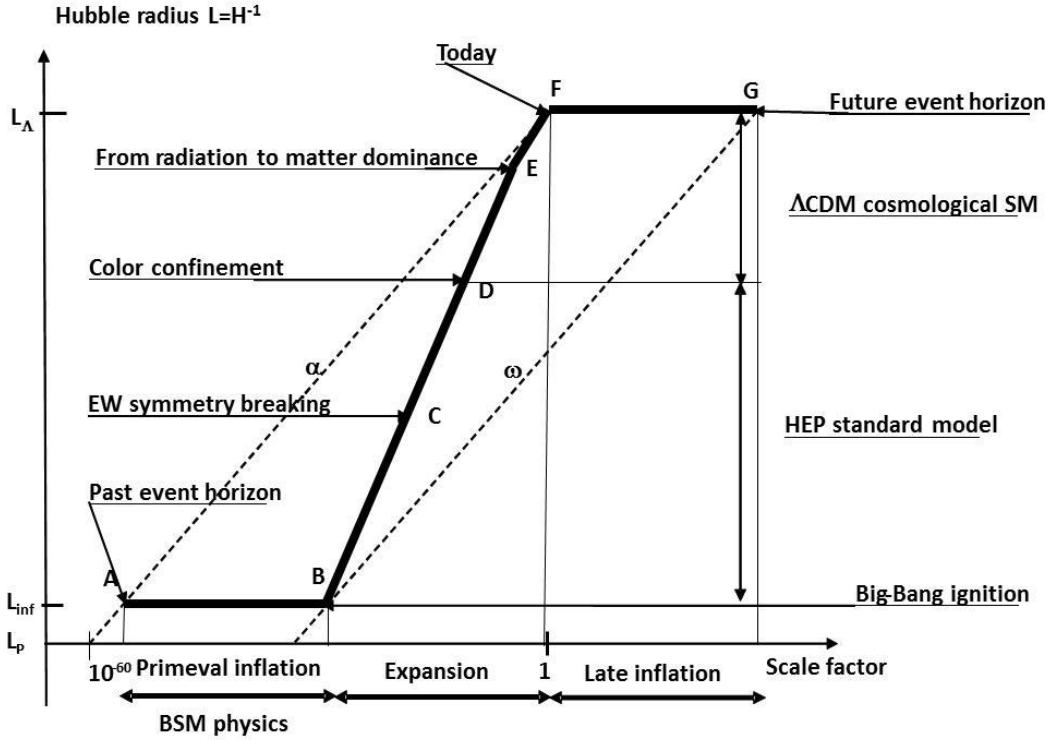


The de Sitter geometry of the quantum vacuum

- Before A (the past event horizon) and after G (the future event horizon, space-time of the observable universe is empty of matter: ρ (matter)=k=0. Inflation is driven by a cosmological constant, an effective CC for the primordial inflation and the observed CC for the late inflation; the geometry is the one of a de Sitter's universe
- **A space-time empty of matter is not the nothingness**: it is the quantum vacuum; i.e. a complex medium in which the quantum fields of the standard model, and possibly a quantized gravitational field are subject to **quantum fluctuations**.
- Such a complex medium can be considered as a perfect fluid with an equation of state $\rho + \mathrm{P} = 0$
- To describe phenomenologically such a medium, we have to include besides the quantum fields of the standard model a (quantum) field to account for gravitation
- In homogeneous and isotropic cosmologies, which are **conformally flat**, the relevant gravitational field is not the metric itsel but a scalar field related to the determinant of the metric, the **dilaton** field ϕ

ΛCDM in the emergent perspective of cosmology

- **A conservation law in ΛCDM**: all the modes that exit from the horizon between A and B go through the horizon between B and F and the re-exit the horizon between F and G
- **This conservation law applies to degrees of freedom, that is to entropy and not to energy**
- This means that **the emergent perspective of gravity applies to ΛCDM**



Dark matter and energy and the quantum vacuum

With the dilaton field ϕ we write the Friedman equation that translates the balancing of the heat fluxes of matter and gravity through the horizon

$$\left\{T_{\mu\nu}(\phi)+T_{\mu\nu}(\text{Matter})\right\} l^{\mu} l^{\nu}=0$$

$$\rho(\phi)+P(\phi)+\rho(\text{Matter})+P(\text{Matter})=0$$

The question is now: what to do with the density and the pressure depending on ϕ?

Two possible answers

$$\left\{T_{\mu\nu}\left(\phi\right)+T_{\mu\nu}\left(\text{Matter}\right)\right\}l^{\mu}l^{\nu}=0$$

$$\rho\left(\phi\right)+P\left(\phi\right)+\rho(\text{Matter})+P\left(\text{Matter}\right)=0$$

The answer of the "relativist"

Dark matter is just an unknown component of Matter

$$P\left(\text{Matter}\right)\approx 0$$

$$\rho\left(\text{Matter}\right)=\rho_{\text{B}}+\rho_{\text{DM}}$$

$$\rho\left(\phi\right)=\rho_{\text{DE}}$$

$$P\left(\phi\right)=-\frac{3H^{2}}{8\pi G_{N}}=-\rho_{\text{c}}$$

$$\rho_{\text{B}}+\rho_{\text{DM}}+\rho_{\text{DE}}=\rho_{\text{c}}$$

The flatness sum rule !

The answer of the "particle physicist"

Dark matter that has no non-gravitational interactions is a component of the quantum vacuum and not of the non-vacuum matter

$$P\left(\text{Matter}\right)\approx 0$$

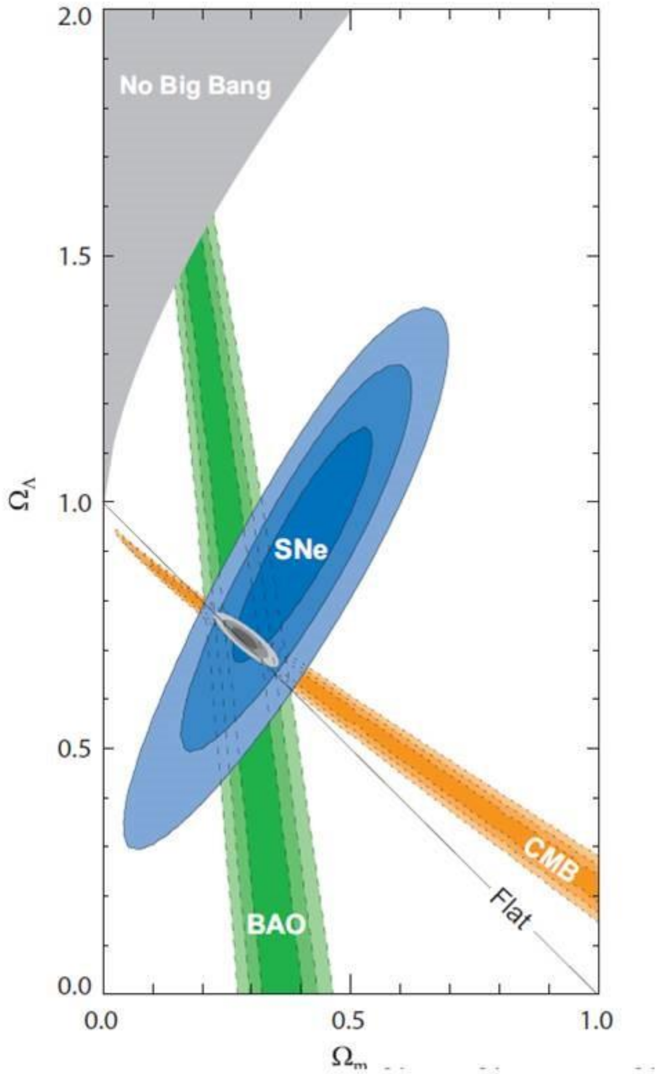
$$\rho\left(\text{Matter}\right)=\rho_{\text{B}}$$

$$\rho\left(\phi\right)=\rho_{\text{DE}}+\rho_{\text{DM}}$$

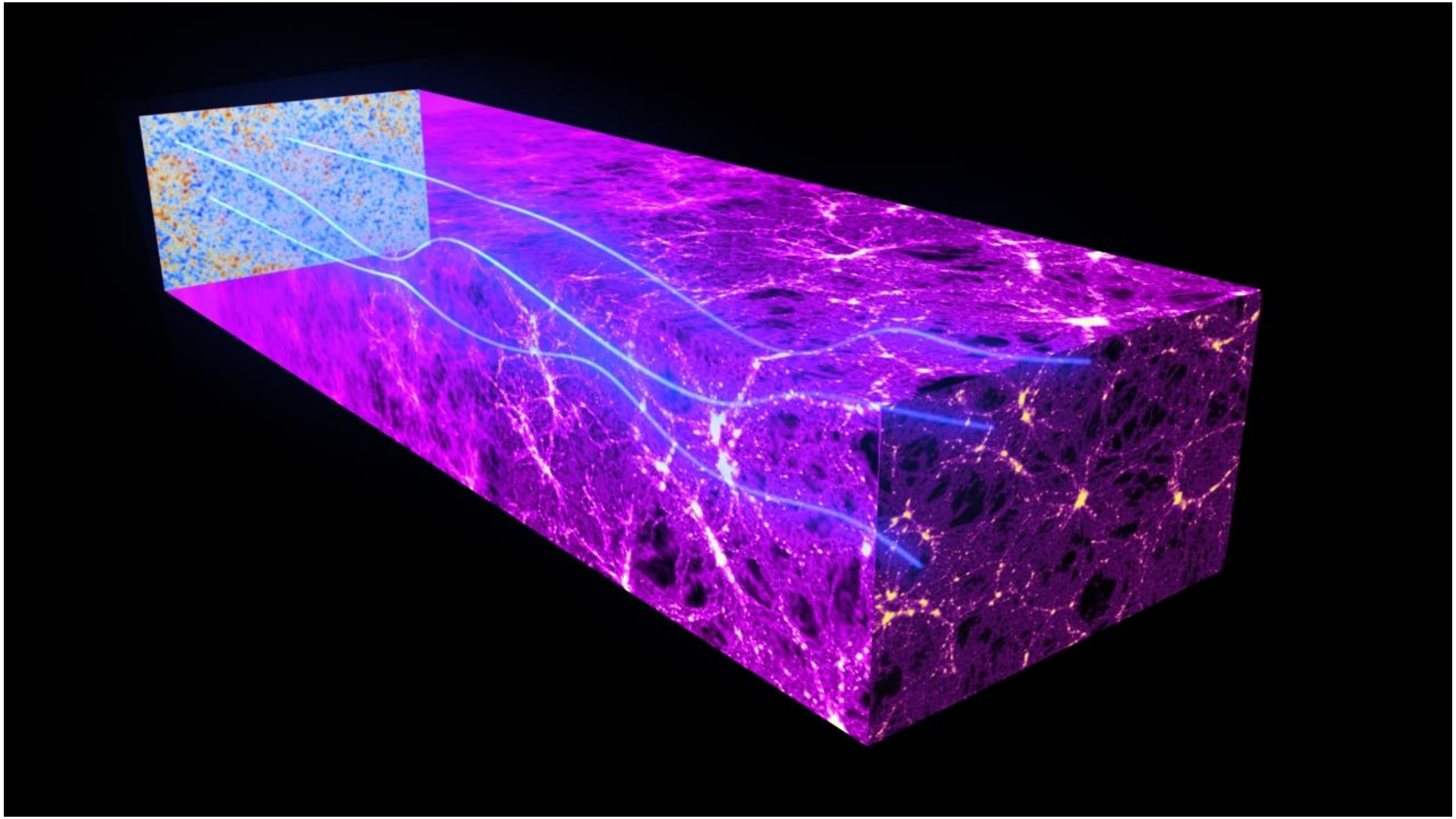
$$P\left(\phi\right)=-\frac{3H^{2}}{8\pi G_{N}}=-\rho_{\Lambda^{eff}}$$

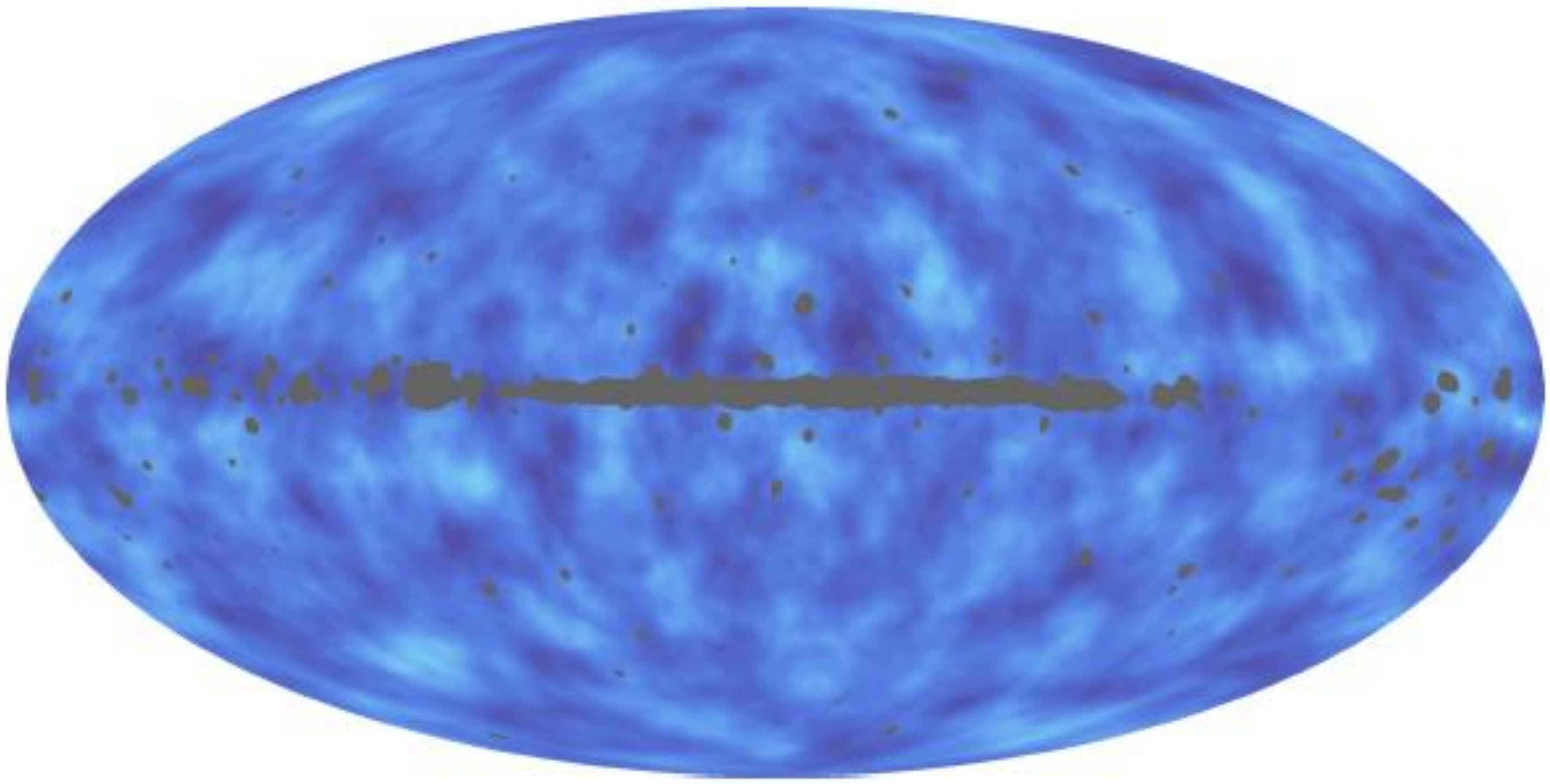
$$\rho_{\text{B}}+\rho_{\text{DM}}+\rho_{\text{DE}}=\rho_{\text{c}}$$

The flatness sum rule !



2.0
1.5
1.0
0.5
0.0
0.0
0.5
1.0
Ω_Λ
Ω_m
No Big Bang
SNe
CMB
Flat
BAO





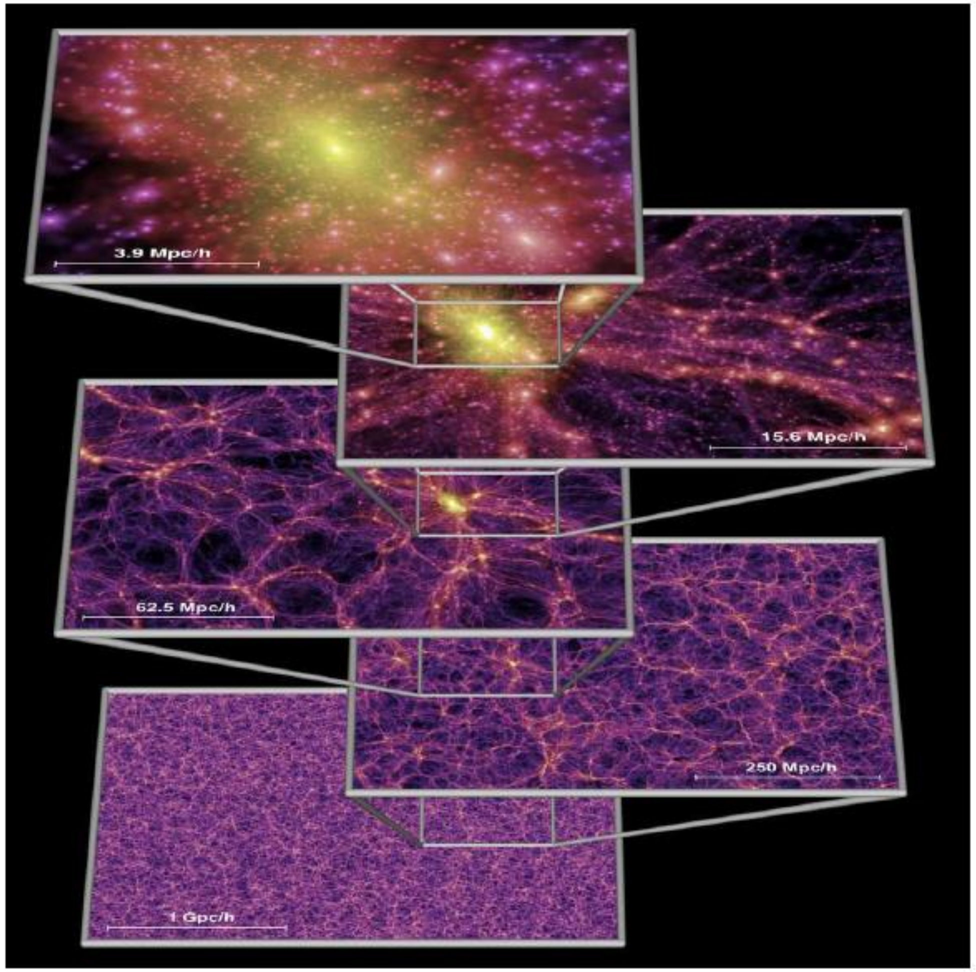
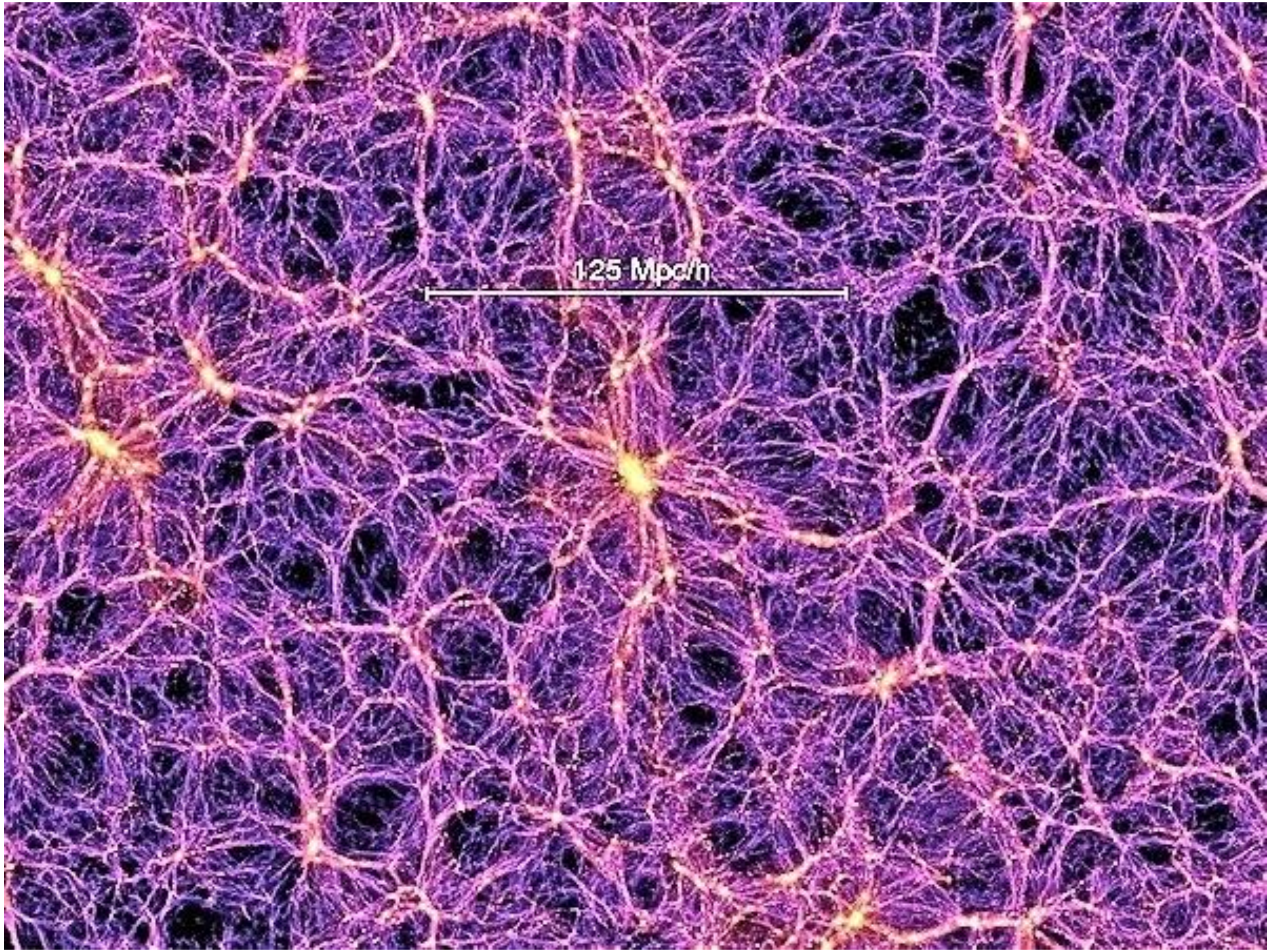
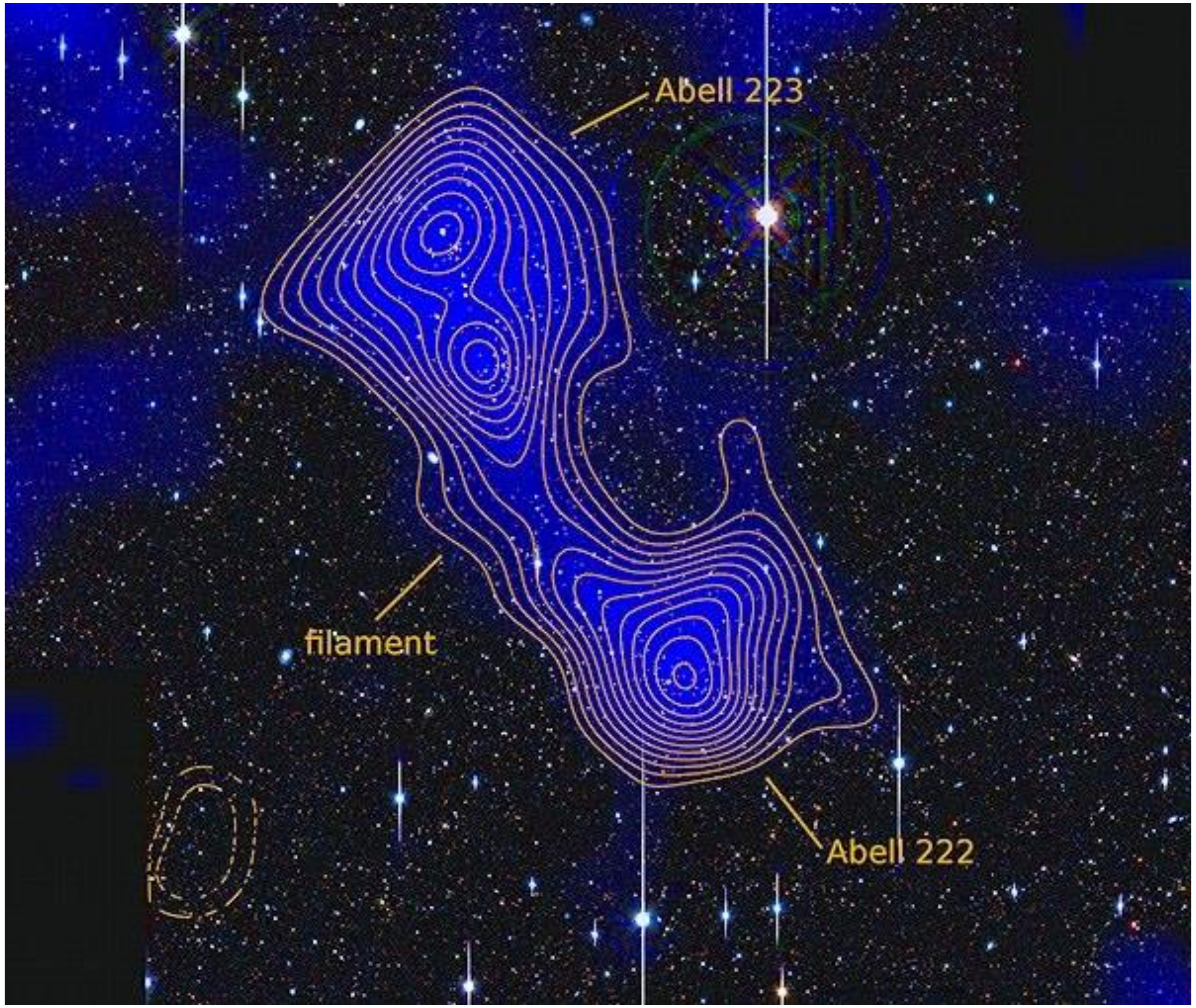
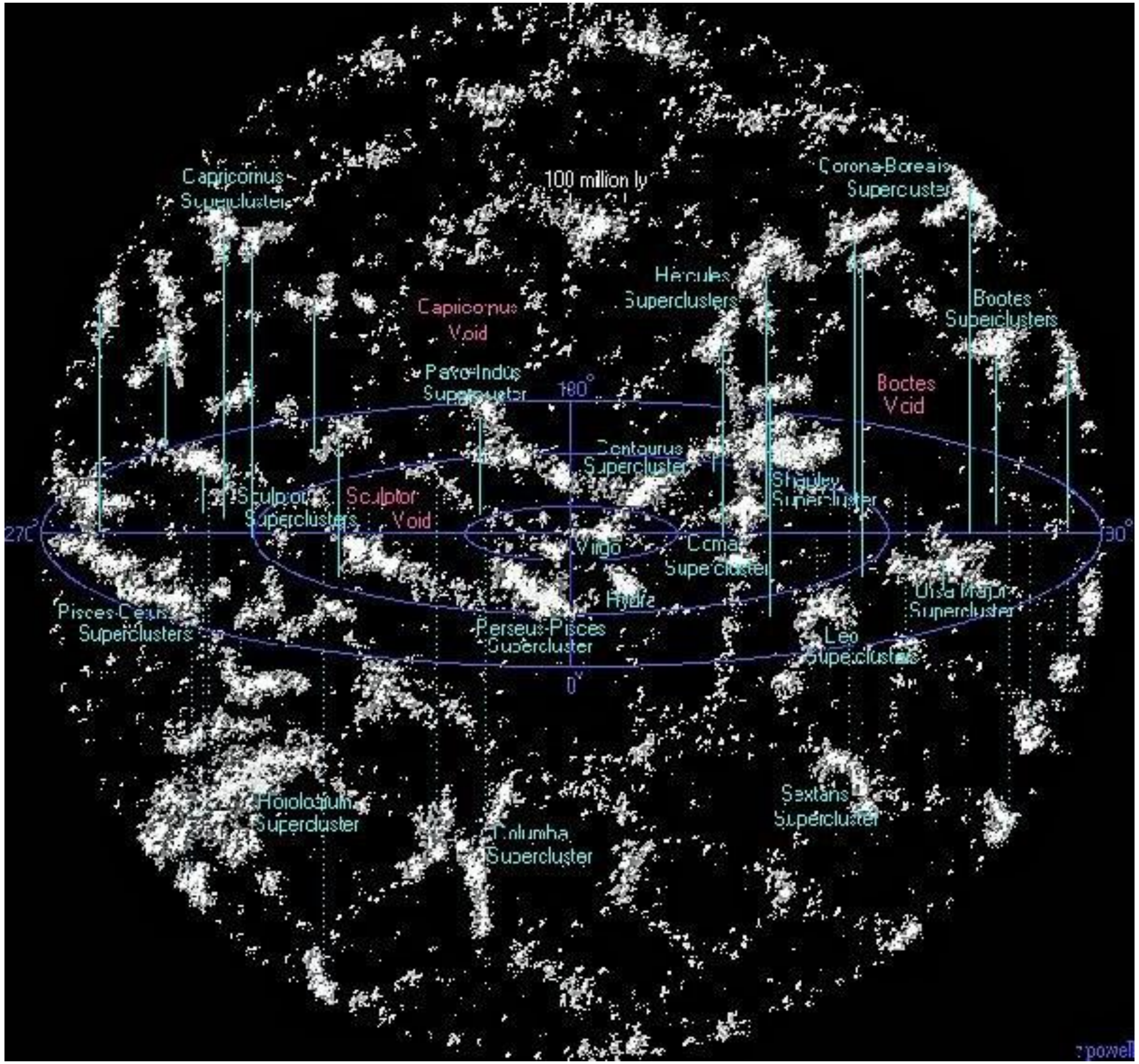


Figure 1: The dark matter density field on various scales. Each individual image shows the projected dark matter density field in a slab of thickness $15\,h^{-1}$Mpc (sliced from the periodic simulation volume at an angle chosen to avoid replicating structures in the lower two images), colour-coded by density


125 Mpc/h


Abell 223
filament
Abell 222


100 million ly
Capricornus Superclusters
Corona Borealis Supercluster
Hercules Superclusters
Capricornus Void
Bootes Superclusters
Pavo-Indus Supercluster
Bootes Void
180°
Centaurus Supercluster
Shapley Supercluster
Sculptor Superclusters
Sculptor Void
270°
Virgo
Coma Supercluster
90°
Hydra
Ursa Major Supercluster
Pisces-Cetus Superclusters
Perseus-Pisces Supercluster
Leo Superclusters
0°
Horologium Supercluster
Sextans Supercluster
Columba Supercluster

- **« D'où l'on peut voir qu'il y autant de différence entre le néant et l'espace vide, que de l'espace vide au corps matériel ; et qu'ainsi l'espace vide tient le milieu entre le matière et le néant. C'est pourquoi la maxime d'Aristote dont vous parlez, "que les non-êtres ne sont point différents", s'entend du véritable néant, et non pas de l'espace vide. »** Réponse de Blaise Pascal au très révérend père Noël, recteur de la Société de Jésus, à Paris, 29 octobre 1647 Pascal, Œuvres complètes, La Pléiade, p 384, ed. 1998

The Mach’s ether

“**There can be no space nor any part of space without gravitational potentials; for these confer upon space its metrical qualities, without which it cannot be imagined at all**. The existence of the gravitational field is inseparably bound up with the existence of space. On the other hand a part of space may very well be imagined without an electromagnetic field; thus in contrast with the gravitational field, the electromagnetic field seems to be only secondarily linked to the ether, the formal nature of the electromagnetic field being as yet in no way determined by that of gravitational ether. From the present state of theory it looks as if the electromagnetic field, as opposed to the gravitational field, rests upon an entirely new formal motif, as though nature might just as well have endowed the gravitational ether with fields of quite another type, for example, with fields of a **scalar potential**, instead of fields of the electromagnetic type.”

A. Einstein, Einstein, *L’éther et la théorie de la relativité* Œuvres choisies, 5, Vol. 5 Science, éthique, philosophie, pp. 81-88, Le Seuil, CNRS, 1991

Reformulation of General Relativity in Accordance with Mach's Principle

Feza Gürsey Annals of Physics 24, 211-242 (1973)

The Mach-Einstein doctrine, which has come to be known as Mach's principle, holds that the basic inertial frame is defined by distant bodies (Mach's fixed stars in the original formulation, now to be understood as the galaxies). According to this view, all inertial effects arise as a consequence of accelerations relative to the system of distant galaxies. In particular, inertia of matter is due entirely to the mutual action of matter. It has been suggested by Einstein that such a mutual action arises from gravitational forces. Then, inertial forces on a body would reduce to gravitational forces exerted by galaxies when the body and the galaxies are in relative accelerated motion. Since, according to this point of view, the need for distinguishing between inertial and gravitational forces disappears, the weaker principle of equivalence follows from the stronger Mach-Einstein principle.

Section IV is devoted to the study of the properties of a special cosmological background uniform in space and time in agreement with the Perfect Cosmological Principle. This is known to be a de Sitter Universe. It is a special conformally flat universe with metric of the form (1.1) with $\lambda=\phi$, ϕ being a definite function of the Lorentz invariant length. It is shown that, although the de Sitter universe cannot contain stable matter**, it may be interpreted as being associate with a uniform distribution of mass scintillations,** that is, unstable masses that give rise to a $\delta^{(4)}(x)$ singularity in the equation determining the metric. In the case of a spatially closed de Sitter world a total mass may then be defined. The metric is expressed in terms of the radius of curvature and the total mass in that case, all the mass coming from mass scintillations.

Finally, an important problem that arises from our discussion concerns the stability and uniqueness of the cosmological structure. We have assumed the universe to have a de Sitter geometry in the first approximation. **This corresponds, as we have seen, to a background of uniformly spread mass scintillations which could be interpreted as virtual pairs of massive particles in a quantum field theoretical picture.** In the second approximation, taking the average contribution of uniformly distributed stable matter into account, we may regard the overall geometry of the universe as conformally flat. Repulsive forces between stable bodies are introduced at this stage. In the third approximation we also allow for the deviations of the Riemannian geometry from the conformally flat structure and thus introduce attractive gravitational forces which are superimposed on an expanding universe. This leads us to question the validity of these successive approximations to the geometry of space-time. Now, in a conformally flat universe we have found the relation (9.6) in which, according to (6.13), part of the total mass M', namely, Nμ comes from stable matter and another part M(0) from the mass scintillations of the de Sitter background. If our approximation is a good one, we must have

$$M(0) \gg N\mu$$

According to McVittie's (39) discussion of recent cosmological data, if we take as *M'* the stable mass in the universe (*M(0)=0*), then the equation (9.6) is off by a factor of about 30. This suggests that

$$N\mu / M(0) \approx 3\%$$

Was the 1917 Einstein's universe so bad?

- **ΛCDM can be interpreted as a quasi-classical, quasi-Newtonian, quasi "perfect", quasi-Machian** Einstein's universe, in which dark matter and energy and a "running" cosmological constant, exactly compensate, at all epochs, the effects of gravity
- Such an equilibrium would be unstable if the universe was static
- But in an expanding universe, such an equilibrium is perfectly admissible, because it is **dynamical**
- Rather that "running", the CC in such an Einstein's universe should be said **"biking"**