COSMOLOGY

Theories and Observations...

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A course of 13 lectures giving historical background to modern cosmology, a critical perspective on the standard approach and some ideas on alternative cosmologies. Both theoretical and observational aspects will be taken into consideration.

Ref. Books: Weinberg "Gravitation and Cosmology" Wiley '72
Narlikar "An Introduction to Cosmology" Cambridge '02
Hoyle, Burbidge, Narlikar "A Different Approach to Cosmology" Cambridge '00.

Newton's static model : "An infinite uniform universe can remain static."

Correspondence with Bentley... 1692–1693

Newton's conclusion : "Such a model is highly unstable."

[Mathematical Papers of Isaac Newton (1976), D.T. Whiteside edited, Cambridge : See Vol. 7, pp. 233, 238]

Later attempts with Newtonian cosmology :

H. Seeliger 1895, Astr. Nachr. cxxxvii, 129.
1896, Münch. Ber. Math. Phys. Kl., 373.

C. Neumann 1896, 'Allgemeine Untersuchungen über das Newtonsche Prinzip der Fernwirkungen' (Leipzig).

W.H. McCrea & E.A. Milne 1934 : 'Newtonian universes and the curvature of space' Qu. J. Math. 5, 73.

The McCrea-Milne models were very similar to the general relativistic models of Friedmann and Lemaitre.

Einstein's approach: 1915 Equations of GR
1917 First application to Cosmology
Preuss. Akad. Wiss. Berlin, Sitzbe
142 (1917)

First discussed the problem within the Newtonian framework

Assume that grav. potential $\rightarrow$ constant at spatial infinity: $r \rightarrow \infty$
$\Rightarrow$ spherically symmetric distribution with $\rho \rightarrow 0$ faster than $1/r^2$

Apply Boltzmann law of gas distribution to deduce $\rho \equiv 0$.

This difficulty disappeared if there existed a cosmological force $F_\lambda = \lambda r$, of repulsion between any two particles of matter separated by distance r.

"The conclusion I shall arrive at is that the field equations of gravitation which I have championed hitherto still need a slight modification, so that on the basis of the general theory of relativity these fundamental difficulties are avoided which have been set forth as confronting the Newtonian theory."

$$R_{ik} - \tfrac{1}{2} g_{ik} R + \lambda g_{ik} = -\kappa T_{ik}$$

In the weak field Newtonian limit the extra term corresponds to $F_\lambda = \lambda r$.

Homogeneous, isotropic and static universe has line element

$$ds^2 = c^2 dt^2 - S_0^2 \left[\frac{dr^2}{1-kr^2} + r^2 \{ d\theta^2 + \sin^2\theta \, d\phi^2 \} \right].$$

S_0 = constant scale factor $k = 0, 1$ or -1.

The field equations yield for $k = +1$

$$\lambda - \frac{3}{S_0^2} = -\frac{8\pi G}{c^2} \rho_0$$

$$\lambda - \frac{1}{S_0^2} = 0.$$

If $k = 0$ or -1, there is no solution.

If $k = 0$, $\lambda = 0$, we get $1/S_0^2 = 0$, $\rho_0 = 0$.

Thus the only non-trivial case is $\lambda > 0$, with

$$S_0 = \frac{1}{\sqrt{\lambda}}, \quad \rho_0 = \frac{\lambda c^2}{4\pi G}.$$

This is the Einstein universe. It appears to give a Machian content to general relativity. → A unique solution in which the geometry of the spacetime background is uniquely fixed by matter content of the universe.

But a counter came to this claim in a few months by de Sitter : 1917 Proc. Akad. Wetensch Amsterdam, 19, 1217.

$$ds^2 = c^2 dt^2 - e^{2Ht} \left[dr^2 + r^2 \{ d\theta^2 + \sin^2\theta \, d\phi^2 \} \right].$$

Motion without matter!

II

Nebular redshifts were being discovered from 1914.

V. M. Slipher (1914) at Lowell Observatory
Lowell Bull. 2, 56

F. G. Pease (1915-1920) continued at Mt Wilson

Milton Humason at Mt Wilson ~ 1927

By early 1930s ... 90 redshifts were measured

1. Most of them were positive

$$z = \frac{\lambda_{observed} - \lambda_{expected}}{\lambda_{expected}} > 0.$$

2. Range was large

$$\sim \text{few } 100 \text{ km/s} < cz < 20000 \text{ km/s}$$

3. Groupings of nebulae with approximately same velocities had been detected by Humason, e.g., by 1932 he had measured z for Virgo, Perseus, Coma and Pegasus as well as Leo clusters.

Curtis used 'Novae' to derive distances of spiral nebulae

Milky Way novae ~ apparent magnitude 5 at distance 10 kly

Some novae in spirals had $m \sim 18$

$$13 = m_1 - m_2 = 5 \log D_1 - 5 \log D_2 = 5 \log(D_1/D_2)$$

$$\frac{D_1}{D_2} = 10^{2.6} \sim 4 \times 10^2$$

$$\Rightarrow D_1 = 4 \times 10^6 \text{ ly}.$$ Overall, for M31 $D \cong 0.5 \times 10^6$ ly

→ Curtis Shapley Debate (1919) Bull. Natl Res. Council Vol 2, pt 3, No. 11

II

A source of confusion! 'S. Andromedae', a 'nova' in M31 was far brighter than an average nova ($M \cong -15.1$ as opposed to $M_{av} \cong -6.1$). 15 years later it was identified as a 'supernova

Hubble used the cepheid variables to estimate the distances of M31 and M33. He found

50 variables including 40 Cepheids in M31

35 cepheids in M33

Shapley had measured 105 in SMC and found

$$m - M = 17.55.$$

For M33 $\quad m - M = 22.1 \Rightarrow D \approx 850,000$ ly

—— M31 $\quad m \cdot M = 22.2 \Rightarrow D \approx 900,000$ ly.

'Zero point' error in SMC would affect all distances. Shapley's values were later revised and M33, M31 distances reduced by 40%.

Van Maanen's work, however consistently pointed to much closer distances, on the basis of proper motions. e.g. at a distance of 10^6 ly an annual proper motion of $\sim 0.01''$ implies a velocity of $\sim 15,000$ km/s.

Huge rotations $\Rightarrow$ spirals would eject matter and would disintegrate in $\sim 10^7$ yrs.

Mt Wilson Observatory did not officially comment on this! Later evaluation of van Maanen's work led to discounting of his claims.

II Who first stated Hubble's law? $\boxed{V = H_0 D}$

Wirtz (1924) showed that radial velocities increased with decreasing diameters of spirals.

Astr. Nacht. 222, 21

Lundmark (1924) found slight correlation between V & D of nebulae.

MNRAS 84, 747

Dose (1927) similar analysis in Astr. Nacht. 229, 157

Lemaitre (1927) actually calculated V/D and found a value not too different from Hubble's two years later.

Ann. de la Societe Scientifique de Bruxelles 47, 49.

Hubble (1929) plot had some distances wrong... would he have found the linear law if the distances were correctly known? Proc. Nat. Acad. Sci (USA) 15, 168

Early value of $H_0 \approx 558 \text{ km s}^{-1} \text{ Mpc}^{-1}$

Write $H_0 = h_0 \times 100 \text{ km s}^{-1} \text{ Mpc}^{-1}$, i.e., $h_0 \cong 5.6$.

Although this value stayed till the early 1950's, it became clear that there were errors of calibration

e.g. there are two different classes of variable stars, the Cepheids & the RR Lyrae. They have different P-L relations.

Also, Hubble had mistaken HII regions for brightest stars.

II

Corrections on both counts increased the distances and so decreased H_0.

By early 1960s Allan Sandage arrived at $h_0 \approx 1$, i.e., $H_0 \cong 100 \text{ km s}^{-1} \text{Mpc}^{-1}$. de Vaucouleurs, however arrived at ~ twice this value.

Sandage (1962) in 'Problems of Extragalactic Research', IAU Symp 15, p. 359 (Reidel / Kluwer).

By early 1970s Sandage further reduced the value to $h_0 \approx 0.5$. de Vaucouleurs value came down close to $h_0 \approx 1$.

de Vaucouleurs { (1976) IAU/CNRS Colloquium
(1980) in Texas Symposium Proceedings.
Proc. of N.Y. Acad. Sci.

Modern value of H_0 (?)

After the HST was launched the 'key' cosmological observation was to determine H_0. Cepheid variables were us[ed]
Wendy Freedman & others find h_0 in the range 0.7 to 0.7[...]
However, Sandage using supernovae still finds h_0 in the range 0.5 and 0.6.

There is still divergence of views!

Early 1930s : Hubble wanted to determine the curvature of space.

$$ds^2 = c^2 dt^2 - S^2(t) \left\{ \frac{dr^2}{1-kr^2} + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2 \right) \right\}$$

Robertson - Walker line element : $k = 0, +1$ or -1.

k enters in the volume - radius relationship for the 3 - spaces.

Can number count of galaxies decide the issue?

Too many galaxies to count!

200 - inch telescope was proposed for the programme.

The telescope was approved & Completed but the programme was abandoned.

Another approach tried during 1960s & 1970s

$\rightarrow$ m-z relation to high z.

To be discussed in a later lecture.

III

Theoretical Models : Based on (i) Weyl's postulate and (ii) Cosmological principle ... applied to General Relativity.

Weyl postulate $\rightarrow$ regularity of motion of matter

Cosmological principle $\rightarrow$ regularity of distribution of matter

Compare with : smooth fluid approximation in hydro-dynamics

Question (1) : Is the approximation valid in a non-linear theory like GR?

$\rightarrow$ Especially in strong-field situation?

Question (2) : Is regularity of motion a good approximation

Now $\longrightarrow$ $V_{random} \approx 300 \text{ km s}^{-1} \ll c = 3 \times 10^5 \text{ km s}^{-1}$.

But $V_{random} \propto \frac{1}{S}$

If S was smaller in the past, is the approximation valid?

e.g. if S were $1/10^3$ of present value, we would have $V_{random} \approx c$.

GR field equations

$$R_{ik} - \frac{1}{2} g_{ik} R = - \frac{8\pi G}{c^4} T_{ik}$$

Weyl postulate + Cosmological principle determine the line element as

$$ds^2 = c^2 dt^2 - S^2(t) \left\{ \frac{dr^2}{1-kr^2} + r^2 (d\theta^2 + \sin^2\theta \, d\phi^2) \right\}$$

$k = 0, +1, -1.$

From 10 non-linear partial differential equations with 4 consistency conditions, the problem is reduced to 2 ordinary differential equations

$$\frac{\dot{S}^2 + kc^2}{S^2} = \frac{8\pi G}{3c^2} T_0^0$$

$$2\frac{\ddot{S}}{S} + \frac{\dot{S}^2 + kc^2}{S^2} = \frac{8\pi G}{c^2} T_1^1 \quad (= T_2^2 = T_3^3).$$

[No off-diagonal elements left.]

What is the consistency condition? This is the matter conservation law:

$$\frac{d}{dt}\left[S^3 T_0^0\right] - 3 T_1^1 S^2 \dot{S} \equiv 0$$

$T_0^0 = \epsilon$ energy density

$-T_1^1 = p$ pressure

What are ϵ and p?

What are the contents of the universe?

Friedmann - Lemaitre : $p = 0$ $\quad \epsilon = \rho c^2$ $\quad$ dust approx.

Gamow : $p = \frac{1}{3}\epsilon$ $\quad$ relativistic / radiation-dominated approximation

Present universe is matter dominated.

$$\rho_{\text{visible}} \approx 4 \times 10^{-31} \, g \, cm^{-3}$$

$$\epsilon_{\text{radiation}} \approx 4 \times 10^{-13} \, erg \, cm^{-3} \ll \rho_{\text{visible}} c^2$$

But $\quad \rho \propto \frac{1}{S^3} \qquad \epsilon \propto \frac{1}{S^4}$

$\Rightarrow$ For small enough S $\quad \rho c^2 \ll \epsilon$.

The status of the cosmological constant :

$$R_{ik} - \frac{1}{2} g_{ik} R + \lambda g_{ik} = -\kappa T_{ik}$$

$$\kappa = \frac{8\pi G}{c^4}.$$

$$2 \frac{\ddot{S}}{S} + \frac{\dot{S}^2 + kc^2}{S^2} - \lambda c^2 = \frac{8\pi G}{c^2} T^1_1$$

$$\frac{\dot{S}^2 + kc^2}{S^2} - \frac{1}{3}\lambda c^2 = \frac{8\pi G}{3c^2} T^0_0$$

Recall : Einstein universe has $S = S_0$ = constant.

$$k = +1 \qquad \lambda \equiv \lambda_c = \frac{1}{S_0^2} \qquad \rho \equiv \rho_0 = \frac{\lambda_0 c^2}{4\pi G}.$$

de Sitter universe has $S \propto e^{H_0 t}$ $\quad H_0$ = constant, $k = 0$

$$\lambda_c = 3H_0^2/c^2 \,, \quad \rho = 0$$

If we demand conservation of energy,

$$T^{ik}{}_{;k} \equiv 0.$$

$$\Rightarrow \qquad \left(R^{ik} - \tfrac{1}{2} g^{ik} R + \lambda g^{ik}\right)_{;k} \equiv 0$$

$$\Rightarrow \qquad \lambda_{;i} = 0 \;\Rightarrow\; \lambda = \text{constant}.$$

Thus Einstein's equations do not permit variation of λ, unless energy conservation law is violated.

λ arises as a Lagrange multiplier in the Hilbert action principle:

$$\delta \int_{\mathcal{V}} R\sqrt{-g}\, d^4x = 0$$

subject to $\displaystyle\int_{\mathcal{V}} \sqrt{-g}\, d^4x$ = constant.

[Constant volume condition!]

If λ has no geometrical significance, write

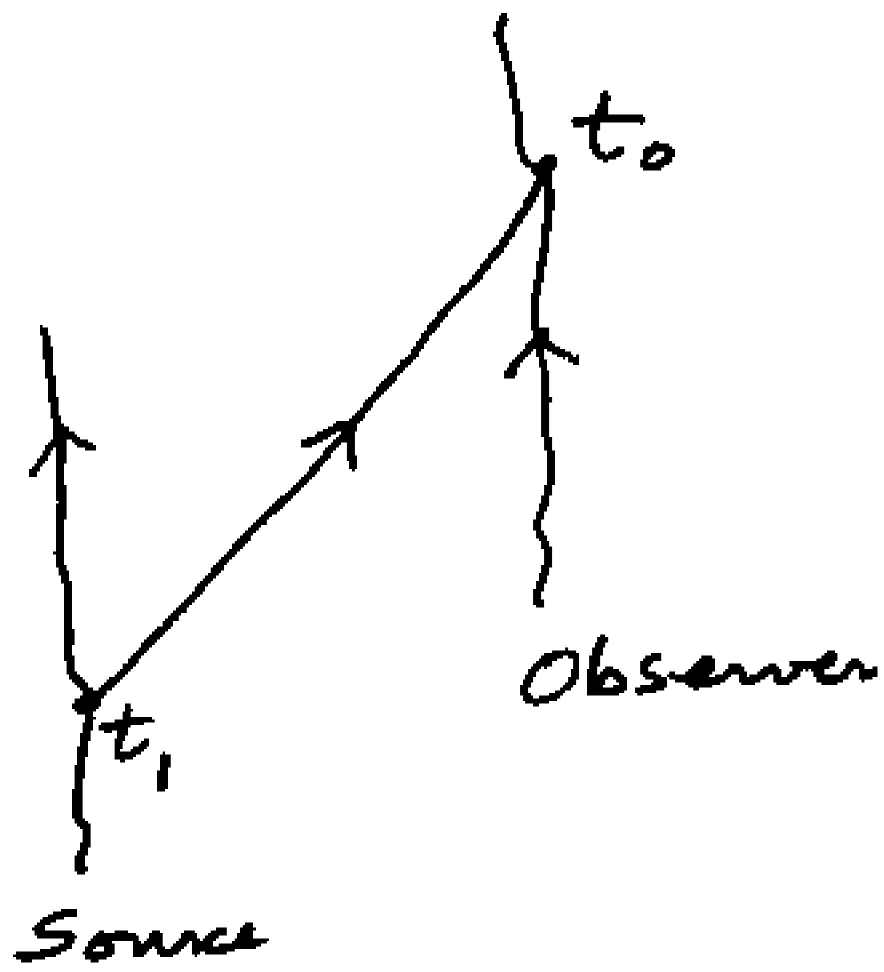
$$R_{ik} - \frac{1}{2} g_{ik} R = -\kappa \left[T_{ik} + \underbrace{\frac{\lambda}{\kappa} g_{ik}}_{\text{dark energy}} \right]$$

If variable λ is needed, use a Lagrangian.

IV

A comment on redshift: Cosmological redshift is given by

$$1 + z = \frac{S(t_0)}{S(t_1)}$$

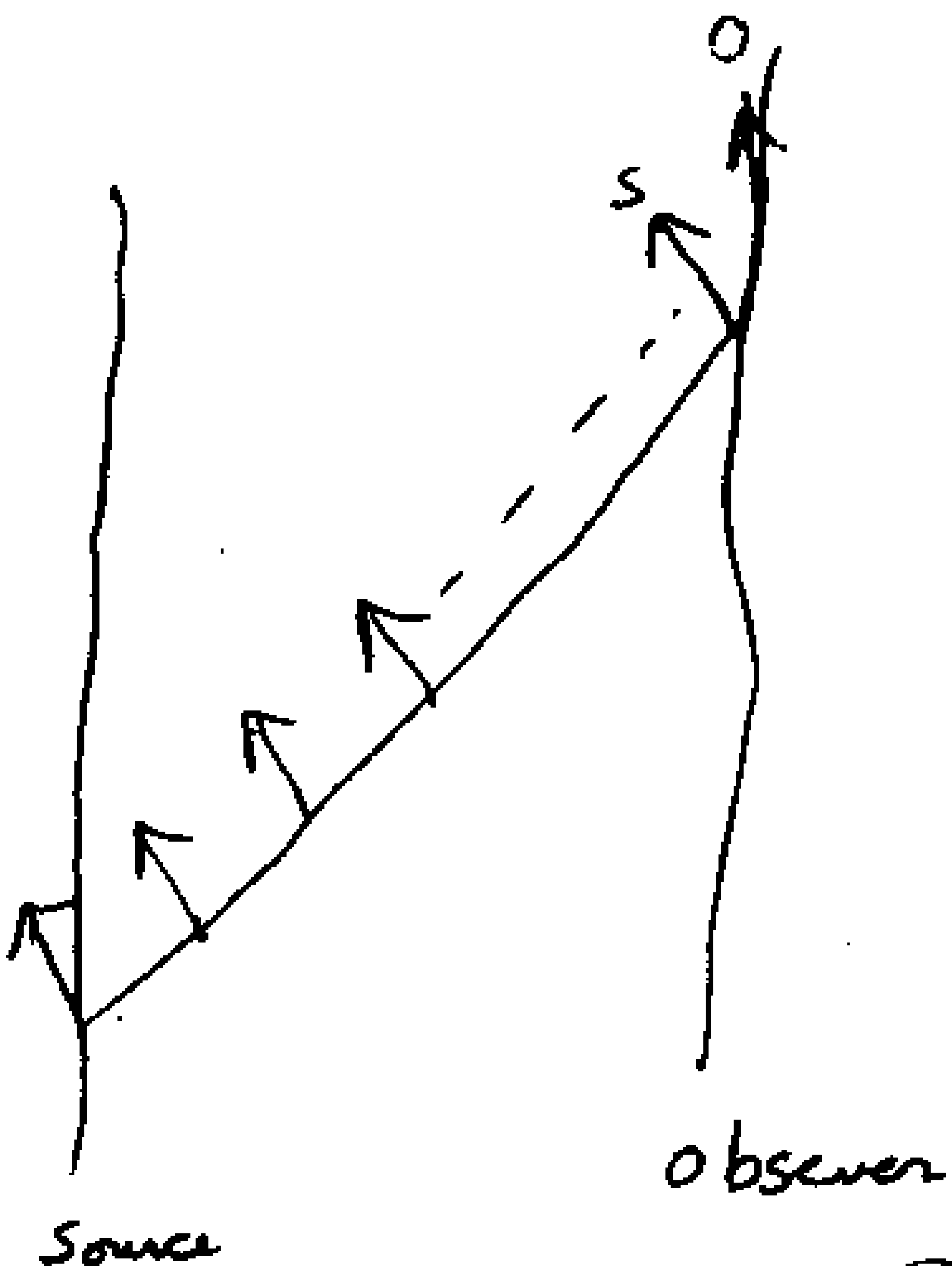


t_1 = time of emission of light

t_0 = time of reception of light

$$z = \frac{\Delta\lambda}{\lambda}$$

Can it be interpreted as a Doppler shift?



Parallelly propagate the tangent vector to the worldline of the source to the observer along the null ray connecting them.

Calculate the Doppler shift of this vector with respect to the velocity vector of the observer.

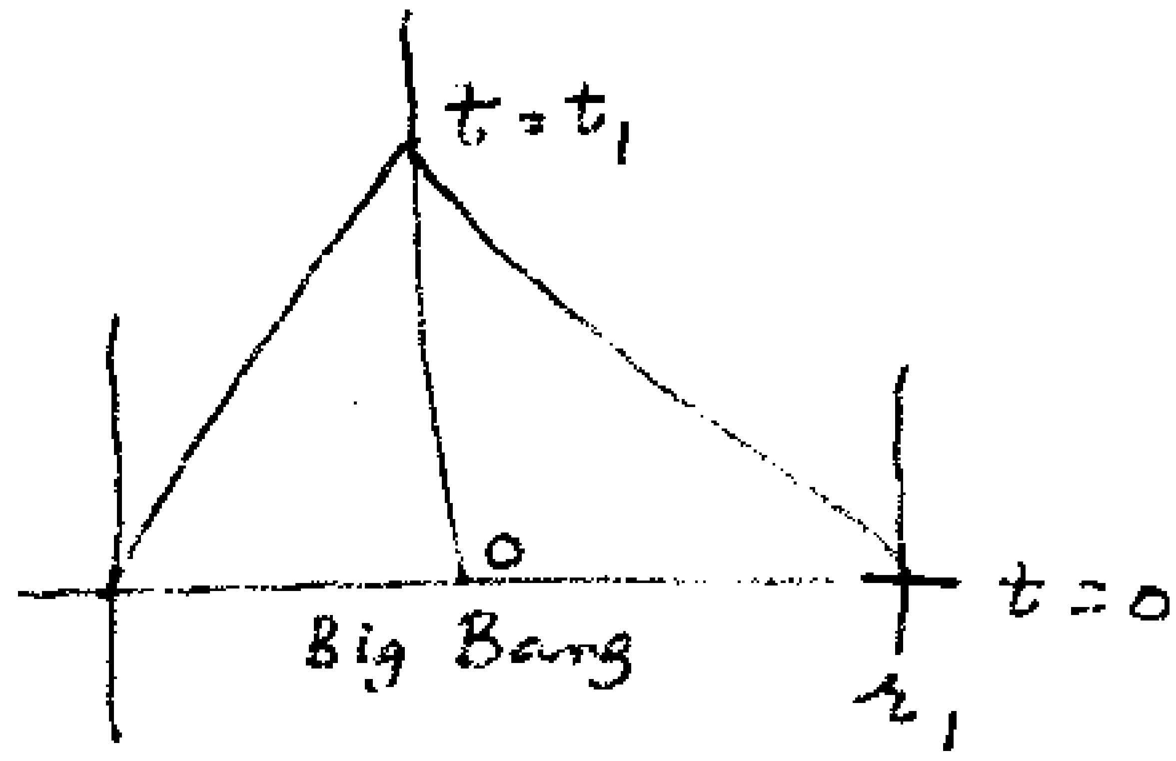
This will be $S(t_0)/S(t_1) - 1 \equiv z$!

IV

Cosmological horizons : There are two types of horizons

(1) Particle horizon limits causal communication from the past ...

$$\int_0^{t_1} \frac{c\,dt}{S(t)} = \int_0^{r_1} \frac{dr}{\sqrt{1-kr^2}}$$



$S(t) = A t^{1/2}$ $\quad k = 0$

$$\int_0^{t_1} \frac{c\,dt}{A t^{1/2}} = \frac{2c}{A} t_1^{1/2} = r_1$$

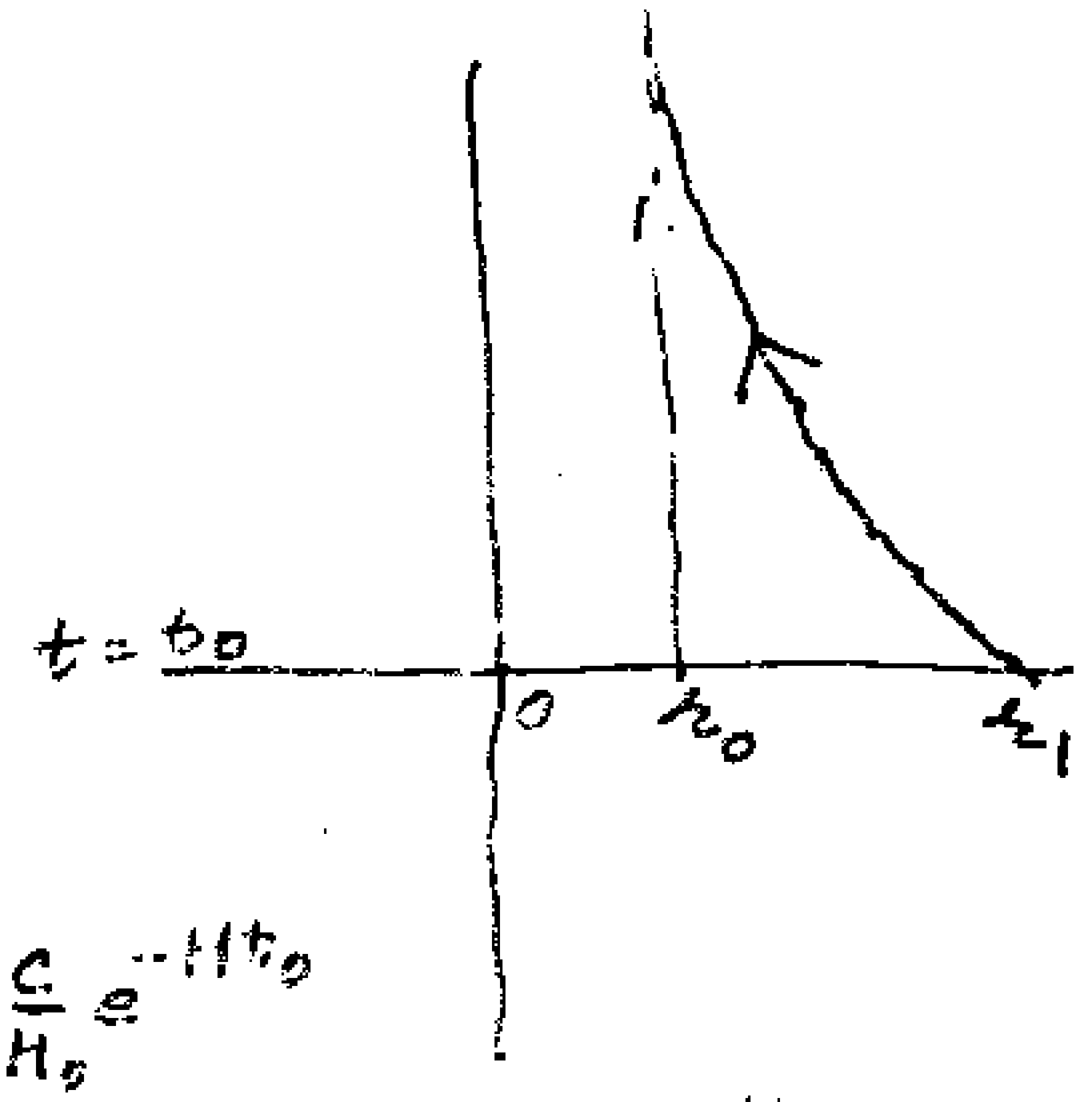
Proper distance at t_1 = $r_1 S(t_1) = \frac{2c}{A} t_1^{1/2} \times A t_1^{1/2} = 2ct_1$

This is the 'particle horizon radius'.

(2) Event horizon limits causal communication to the future ...

$S(t) = e^{H_0 t}$ $\quad k = 0$

$$\int_{t_0}^{\infty} \frac{c\,dt}{S(t)} = r_1 - r_0$$



$$r_1 - r_0 = \int_{t_0}^{\infty} \frac{c\,dt}{e^{H_0 t}} = \frac{c}{H_0} e^{-H t_0}$$

For signal to reach $r_0 = 0$, $r_1 = \frac{c}{H_0} e^{-H t_0}$

in the limiting case; i.e., in general $r_1 < \frac{c}{H_0} e^{-H t_0}$.

Proper distance at t_0 = $r_1 e^{H t_0} = \frac{c}{H_0}$. → "Event Horizon Radi.

IV

A common mistake is confusion between the two types of horizons.

Very often we use the notion of 'Hubble Radius' which has nothing to do with the global property of spacetime.

$$H = \frac{\dot{S}}{S} \qquad \text{Hubble radius: } c/H.$$

This length characterizes the typical geometrical size of the universe. It has <u>no</u> significance with causality

Some typical quantities of relevence to Friedmann models

$$H = \frac{\dot{S}}{S} \qquad H(t_0) = H_0 = 100\, h_0\, km\, s^{-1}\, Mpc^{-1}$$

$$q_0 = -\frac{1}{H_0^2}\frac{\ddot{S}}{S} \qquad \text{deceleration parameter}$$

$$\rho_0 = \frac{3H_0^2}{8\pi G} \quad \text{closure density} \quad (k=0 \text{ model})$$

$$\rho = (\rho/\rho_0) \times \rho_0 = \Omega_0 \rho_0$$

$$\Omega_\Lambda = \frac{\lambda c^2}{3H_0^2} \quad (\equiv \Lambda)$$

(Note: λ = constant, but Ω_Λ varies with epoch.)

Apply to field equations at present epoch:

$$H_0^2 + \frac{kc^2}{S_0^2} - \frac{1}{3}\lambda c^2 = H_0^2 \Omega_0$$

$$(1-2q_0)H_0^2 + \frac{kc^2}{S_0^2} - \lambda c^2 = 0$$

$\underline{\lambda = 0}$

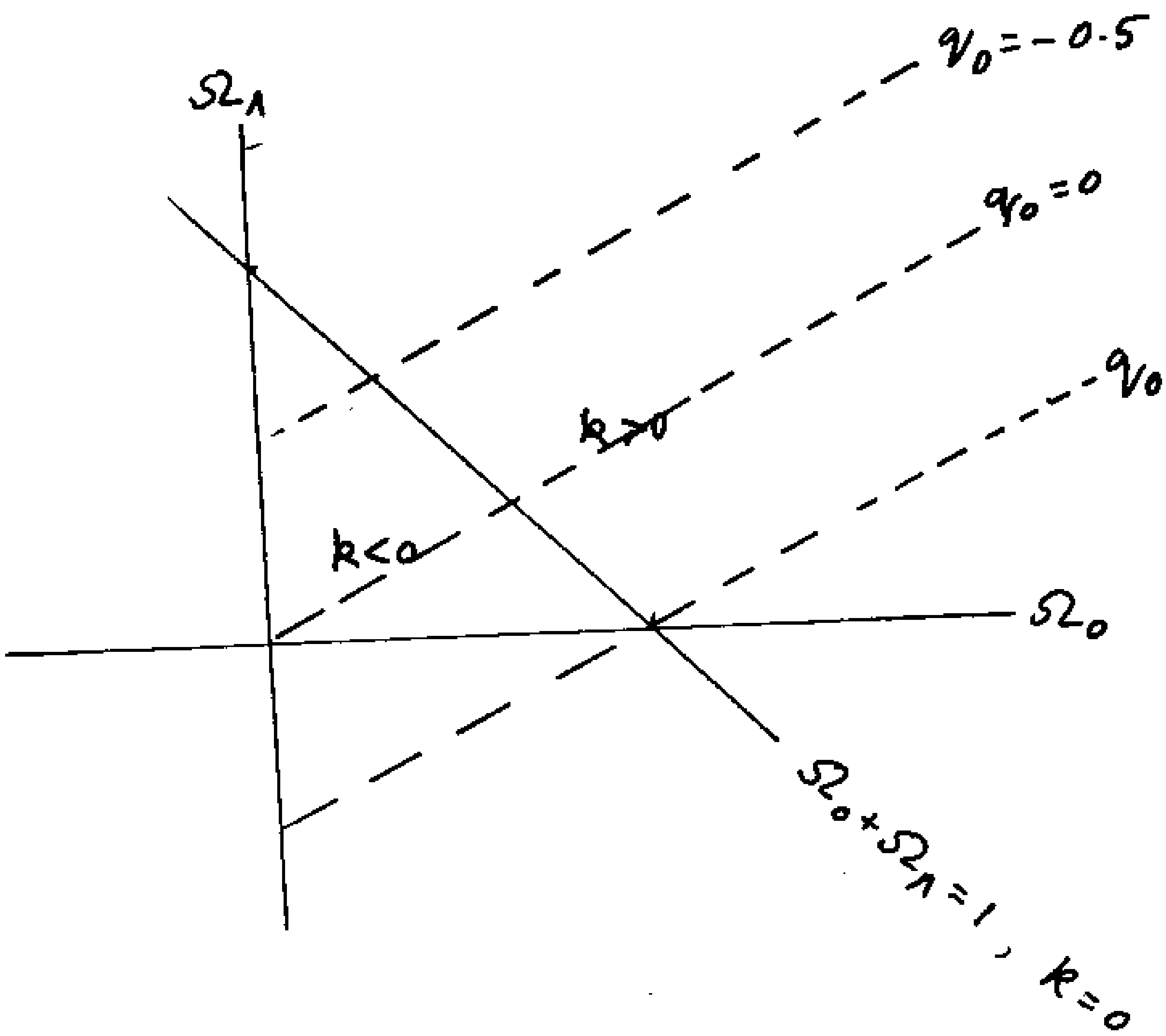
$$\frac{kc^2}{S_0^2} = (2q_0 - 1)H_0^2. \Rightarrow \begin{array}{lll} k=+1, & q_0 > \frac{1}{2} & , \Omega_0 > 1 \\ k=-1, & q_0 < \frac{1}{2} & , \Omega_0 < 1 \\ k=0, & q_0 = \frac{1}{2} & , \Omega_0 = 1. \end{array}$$

$$\Omega_0 = 2q_0$$

$\underline{\lambda \neq 0}$

$$\Omega_0 = 2q_0 + 2\Omega_\Lambda$$

$$k = 0 \Rightarrow \quad \Omega_0 + \Omega_\Lambda = 1$$



$q_0 > 0$ decelerating universe

$q_0 < 0$ accelerating universe

Luminosity distance

Spectral function at source : $dL = L\, I(\lambda)\, d\lambda$

$$\int_0^\infty I(\lambda)\, d\lambda = 1$$

Total luminosity of the source $= L$

Wavelength range at source $= (\lambda_1, \lambda_1 + \Delta\lambda_1)$

" " " observer $= (\lambda_0, \lambda_0 + \Delta\lambda_0)$

$$\lambda_0 = (1+z)\lambda_1 \qquad \Delta\lambda_0 = (1+z)\Delta\lambda_1 .$$

Energy leaving the source in time Δt_1 in $(\lambda_1, \lambda_1 + \Delta\lambda_1)$

$$L\, I(\lambda_1)\Delta\lambda_1 \Delta t_1 = L\, I\left(\frac{\lambda_0}{1+z}\right) \frac{\Delta\lambda_0}{1+z} \cdot \frac{\Delta t_0}{1+z} .$$

[Time interval Δt_1 at source corresponds to $\Delta t_0 = (1+z)\Delta t_1$ at the observer.]

This energy is made of quanta of frequency $\sim c/\lambda_1$. Each quantum has energy ch/λ_1. [This is changed to ch/λ_0, i.e., a reduction by a factor $(1+z)$, at the observer.]

Number of photons emitted in Δt_1

$$N = \frac{L\, I(\lambda_1)\Delta\lambda_1 \Delta t_1}{ch/\lambda_1} = L\, I\left(\frac{\lambda_0}{1+z}\right) \cdot \frac{\Delta\lambda_0}{1+z} \cdot \frac{\Delta t_0}{1+z} \cdot \frac{\lambda_1}{ch}$$

$$= \frac{L\lambda_0}{ch} \cdot \frac{1}{(1+z)^3}\, I\left(\frac{\lambda_0}{1+z}\right) \cdot \Delta\lambda_0 \Delta t_0$$

Number received per unit area $\bar{L}$ to the flux at the source

$$\frac{L\lambda_0}{ch} \cdot \frac{1}{(1+z)^3} I\left(\frac{\lambda_0}{1+z}\right) \Delta\lambda_0 \cdot \Delta t_0 \cdot \frac{1}{4\pi r_1^2 S^2(t_0)}$$

The energy carried by each photon is $\frac{hc}{\lambda_0}$. Hence the flux received per unit area per unit wavelength range is

$$l(\lambda_0) = L \cdot \frac{1}{(1+z)^3} \cdot I\left(\frac{\lambda_0}{1+z}\right) \cdot \frac{1}{4\pi r_1^2 S^2(t_0)} .$$

Total (bolometric) flux is

$$l = \frac{L}{4\pi r_1^2 S^2(t_0)(1+z)^2} .$$

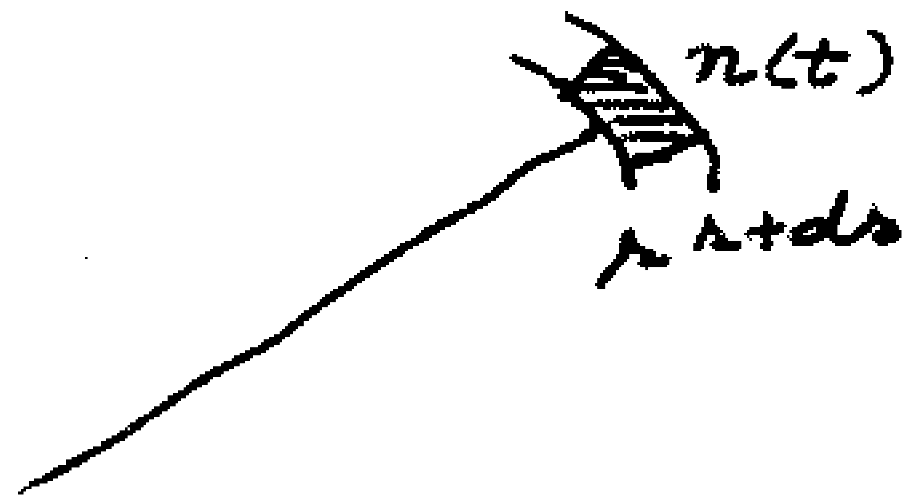
K-correction: An important correction at large redshifts is the K-correction, i.e., the correction because of shift of the spectrum.

e.g. take $z = 2$. At observer the wavelength is, say, 450 nm. This corresponds to a wavelength 150 nm at source. Thus we need to know the UV-spectrum of the source.

Stebbins-Whitford Effect.

V

Source counts: The original Hubble programme of counting galaxies was based on counting galaxies upto varying distances, i.e., redshifts.



No in unit coordinate volume at epoch t $= n(t)$

No. upto $r \leq r_1$

$$N(r_1) = \int_0^{r_1} \frac{4\pi r^2 dr}{\sqrt{1-kr^2}} \, n(t).$$

$$\int_t^{t_0} \frac{c\,dt'}{S(t')} = \int_0^{r} \frac{dr'}{\sqrt{1-kr'^2}}$$

Galaxies (Hubble in the 1930s)

Radio sources — Ryle at Cambridge, Mills at Parkes / Sydney, Bolton at Caltech — 1950-65

Later surveys showed the indefinitive nature of the conclusion.

Galaxies (Computer counting at faint magnitudes).

Learn about possible evolution than geometry of the model.

III The steady state theory : Bondi & Gold (1948), Hoyle (1948)

MNRAS 108, 252 MNRAS, 108, 372

The Bondi-Gold approach was via the Perfect Cosmological Principle

① Rationale ② Deductions

The PCP was used to deduce (i) the universe is expanding (ii) the universe has constant density (iii) there is a steady creation of matter (iv) the spacetime has de Sitter line element.

There was no dynamical theory. This was provided by Hoyle's approach. This was based on (i) arguing that the big bang 'creation' of matter is inconsistent with the law of conservation of matter & (ii) that the singularity is inconsistent with field equations

$$R^{ik} - \frac{1}{2} g^{ik} R = -K \left\{ \underset{(\text{matter})}{T^{ik}} + \underset{(\text{creation terms})}{T^{ik}} \right\}$$

M.H.L. Pryce gave an elegant derivation of

$$\underset{\text{creation}}{T^{ik}} = -f \left\{ C^i C^k - \frac{1}{2} g^{ik} C^l C_l \right\}$$

for $C_i \equiv \partial C / \partial x^i$, C a scalar field.

VI Objections by physicists to the C-field during 1950s & 1960s:

① Scalar fields do not exist

② Negative energy fields do not exist

③ Non-conservation of baryons is not possible.

Today physicists use negative energy scalar fields and believe that baryon number is not conserved at very high energies.

Some objections by astronomers during 1950s & 1960s:

① Stebbins Whitford effect is inconsistent with 'steady state'.

② Source counts (of radio sources) show evolution.

③ The universe is slowing down, whereas a steady state universe accelerates.

Today none of these objections stand!

However, the steady state theory could not explain the evidence during the 1960s & 1970s on (i) the observed abundance of light nuclei like ^4He, ^2H, and (ii) the observed isotropic microwave background.

VIII

The early universe

T^{ik} at present is matter dominated.

$$\rho_{matter} \gtrsim 10^3 \rho_{radiation}$$

However
$$\underset{matter}{T^{ik}} = \rho_m u^i u^k$$

$$\underset{radiation}{T^{ik}} = \frac{4}{3}\rho_r u^i u^k - \frac{1}{3}\rho_r g^{ik}$$

$$\underset{m}{T}{}^{ik}{}_{;k} = 0 \Rightarrow \rho_m \propto \frac{1}{S^3}$$

$$\underset{r}{T}{}^{ik}{}_{;k} = 0 \Rightarrow \rho_r \propto \frac{1}{S^4}$$

$\Rightarrow$ For sufficiently small S, $\rho_r > \rho_m$.

The early universe $(S < S_0/1000)$ was radiation-dominated

Random motions $\propto \frac{1}{S}$

$$\text{Pressure} \sim \rho v_r^2 \sim \frac{1}{S^5}$$

As we probe more & more into the past, we should see the pressure of matter dominate. As v_r increases, matter becomes relativistic... i.e., there is no difference between matter & radiation.

Photonic universe : photons in thermodynamic equilibrium

$$\rho_r = aT^4 \propto \frac{1}{S^4} \rightarrow \rho_r = \frac{B}{S^4}$$

$$3\,\frac{\dot{S}^2 + kc^2}{S^2} = 8\pi G\rho_r = 8\pi G a T^4$$

Assume to start with that $|kc^2| << \dot{S}^2$.

$$\Rightarrow \quad \frac{\dot{S}^2}{S^2} = \frac{8\pi G\rho_r}{3} = \frac{8\pi G B}{3S^4}$$

$$\Rightarrow \quad S\dot{S} = \text{constant} \Rightarrow S^2 \propto t \Rightarrow \dot{S}/S = \frac{1}{2t}.$$

$$\frac{1}{4t^2} = \frac{8\pi G\rho_r}{3} = \frac{8\pi G a T^4}{3}$$

$$\Rightarrow \quad \underbrace{T = \left(\frac{3}{32\pi G a}\right)^{1/4} t^{-1/2}.}_{\text{Time - temperature relationship!}}$$

$$T = \left[1.52 \times 10^{10}\, t_{\text{second}}^{-1/2}\right] K$$

How does this change with relativistic particles?

The black body law for photons is modified for massive fermions & bosons as follows. Call it particle A.

E_A = energy of a particle of mass m_A

P_A = momentum of a particle of mass m_A

$$E_A^2 = P_A^2 c^2 + m_A^2 c^4.$$

g_A = number of spin states of particle A.

$n_A(P)$ = number density of particle of momentum P per unit momentum range.

$$= \frac{g_A}{2\pi^2\hbar^3} \times \frac{p^2}{\exp\left\{\frac{E_A(P)-\mu_A}{kT}\right\} \pm 1} \qquad \begin{array}{l} + \text{ fermions} \\ - \text{ bosons} \end{array}$$

T = temperature of the distribution

μ_A = chemical potential of the distribution

μ_A can be set equal to zero for photons

μ_A can be neglected for particles/antiparticles.

$N_B/N_\gamma \ll 1$.

Total over all momenta obtained by integration

N_A = number density of species A

$$= \int_0^\infty n_A(P)\, dP$$

energy density $\epsilon_A = \int_0^\infty E_A(P)\, n_A(P)\, dP$

pressure $p_A = \frac{1}{3}\int_0^\infty \frac{p^2}{E_A(P)} \cdot n_A(P)\, dP$

entropy $s_A \cong \frac{1}{T}\{p_A + \epsilon_A\}$

In the relativistic limit for particle A

$$\epsilon_A \gg m_A c^2.$$

$$T \gg T_A = \frac{m_A c^2}{k} \quad \text{High temperature case.}$$

At $T \lesssim T_A$ we have low temperature approximation.

$$N_A = \frac{g_A}{\hbar^3}\left(\frac{m_A kT}{2\pi}\right)^3 \exp\left(-\frac{T_A}{T}\right)$$

$$\epsilon_A = m_A N_A, \quad p_A = N_A kT, \quad s_A = \frac{m_A N_A c^2}{T}.$$

At high temperatures

$$\epsilon_A = \begin{cases} \frac{1}{2} g_A \epsilon_\gamma & \text{for bosons} \\ \frac{7}{16} g_A \epsilon_\gamma & \text{for fermions} \end{cases} \qquad \epsilon_\gamma = \frac{\pi^2 (kT)^4}{15\hbar^3 c^3} = aT^4$$

For a mixture of bosons & fermions

$$\epsilon = \rho_r c^2 = \frac{1}{2} g a T^4$$

$$g = g_b + \frac{7}{8} g_f$$

g_b = total bosonic degrees of freedom

g_f = total fermionic degrees of freedom

e.g. $\{\gamma, e^-, e^+, \nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu\}$

$g_b = 2 \quad g_f = 2+2+1+1+1+1 = 8 \Rightarrow g = 9.$

A general line of argument used:

A process works at a significant rate in the primordial universe, provided it exceeds the rate of expansion of the universe.

Processes which took place in the primordial universe:

a) Neutrino decoupling $\nu_e + \mu^- \leftrightarrow \nu_\mu + e^-$

Neutrino reaction rate $\eta \propto T^5 \exp\left(-\frac{T_\mu}{T}\right)$

[cross section $\propto T^2$, electron density $\propto T^3$, muon density $\propto \exp(-T_\mu/T)$.]

Reaction rate to be compared with expansion rate

$$H^2 = \frac{\dot{S}^2}{S^2} \propto \epsilon \propto T^4$$

$$\frac{\eta}{H} \cong T_{10}^3 \exp\left(-\frac{1}{T_{12}}\right)$$

$$\left[T_n = \frac{T}{10^n}\right]$$

Neutrinos decouple at $T_{10} \approx 1$.

But $T_\nu \propto \frac{1}{S}$ even after decoupling.

b) Pair annihilation

$$e^- + e^+ \rightarrow \gamma + \gamma \qquad \text{As } T \text{ falls} \lesssim T_e \sim 6\times10^9\,K$$

Energy of $e^\pm$ is dumped into photons.

$\Rightarrow$ Photon temperature rises.

Earlier $\quad T_\gamma = T_\nu$

Later, after all $e^\pm$ are annihilated $\quad \frac{T_\gamma}{T_\nu} = \left(\frac{11}{4}\right)^{1/3} \sim 1.4$

At present $T_\gamma \sim 2.7K$

So we expect $T_\nu \sim 1.9\,K$.

THIS IS A DEFINITE PREDICTION!

c) n-p ratio and ^{4}He abundance

$$N_p = \frac{2}{\hbar^3}\left(\frac{m_p kT}{2\pi}\right)^{3/2} \exp\left(-\frac{T_p}{T}\right)$$

$$N_n = \frac{2}{\hbar^3}\left(\frac{m_n kT}{2\pi}\right)^{3/2} \exp\left(-\frac{T_n}{T}\right)$$

$$\frac{N_n}{N_p} \sim \exp\frac{T_p - T_n}{T} \sim \exp\left(-\frac{1.5}{T_{10}}\right)$$

~ 1 at $T \gtrsim 10^{12}\,K$

$\sim \frac{5}{6}$ at $T \sim 10^{11}\,K$

$\sim \frac{3}{5}$ at $T \sim 3\times10^{10}\,K$

^{4}He abundance is determined by n/p ratio.

Suppose there are N protons and Nr neutrons, $0<r<1$.

1 He atom has $2n$ & $2p$.

Hence Nr neutrons will make $\frac{Nr}{2}$ He atoms.

Mass of $\frac{Nr}{2}$ He atoms $= 4 \times \frac{Nr}{2} = 2Nr$

Total mass $= Nr + N = N(1+r)$

Helium mass fraction $Y = \frac{2Nr}{N(1+r)} = \frac{2r}{1+r}$

For $Y = 1/4$, $r = 1/7$.

$$r = \frac{N_n}{N_p} = \exp\left(-\frac{1.5}{T_{10}}\right)$$

$H^2 \propto g T^4$ $\quad g =$ d.f. of relativistic species.

Reaction rate for $n \leftrightarrow p$ via β-decay & other weak interactions $= \eta \propto T^4$

Let $\eta = H$ at $T = T_*$

Then $T_*^2 \propto g^{1/2}$

$$r = \exp\left(-\frac{1.5}{T_{10*}}\right). \qquad Y = \frac{2r}{1+r}$$

$$\delta Y = -\frac{r \ln r}{(1+r)^2} \frac{\delta g}{g}$$

For l lepton families $g = \frac{7}{8}(4+2l)+2$

If there are l leptons of e & ν pairs, an increase of l will affect Y, increasing it to $Y+\delta Y$.

From $l=2$ to $l=3$, $\delta g/g \simeq \frac{1}{5}$ For $Y = 1/4$ $\sim = \frac{1}{7}$,

$\delta Y = 0.02$

Comparison with observations suggests $l=3$.

This agrees with accelerators which also give $l=3$.

d) Formation of deuterium

Reactions giving a nucleus Q^A_Z

$$B_Q = \left[Z m_p + (A-Z) m_n - m_Q\right] c^2 \qquad N_N = \text{number of nucleons}$$

$$X_n = \frac{N_n}{N_N} \qquad X_p = \frac{N_p}{N_N} \qquad X_Q = \frac{N_Q A}{N_N}$$

$$X_Q = \frac{1}{2} g_Q A^{5/2} X_p^Z X_n^{A-Z} \xi^{A-1} \exp\left(\frac{B_Q}{kT}\right)$$

$$\xi = \frac{1}{2} N_N \left(\frac{mkT}{2\pi\hbar^2}\right)^{-3/2}$$

For deuterium

$$X_D = \frac{3}{\sqrt{2}} X_p X_n \xi \exp\left(\frac{B_D}{kT}\right)$$

III

Reactions proceed to bind n & p into 'D' $\equiv {}^2H$ if T drops to less than 10^9 K.

$$p + n \leftrightarrow D + \gamma$$

$$D + D \longleftrightarrow {}^3He + n \longleftrightarrow {}^3H + p$$

$${}^3H + D \leftrightarrow {}^4He + n$$

If lot of n & p are present, lot of D is formed. Then the following reactions proceed fast to annihilate it!

$\Rightarrow$ n, p density should not be too high.

Present density of baryonic matter $= \rho_{B0}$. Define

$$\eta = \left\{ \frac{\rho_{B0}}{1.97 \times 10^{-26} \, g \, cm^{-3}} \right\} \times \left(\frac{2.7}{T_0} \right)^3$$

$$\rho_B = \eta \, T_9^3 \qquad T_9 < 3$$

For appreciable deuterium to be formed we need $\eta <$ a few times $\times 10^{-4}$. That is, for $X_D \sim 2 \times 10^{-5}$, say,

$$\Omega_B h_0^2 \lesssim 0.02 \qquad \rho_{B0} \sim 4 \times 10^{-31} \, g \, cm^{-3}$$

Problem of Dark Matter!

Microwave background : Alpher & Herman (Nature 162, 774, 1948)
Penzias & Wilson (ApJ., 142, 419, 1965)

Earlier estimate by Eddington in the 1920s from estimate of starlight from the Galaxy $\rightarrow T_0 \sim 2\text{-}3\,K$

Earlier detection by McKeller in 1941, Pub. Dom. Astrophys. Obs. Victoria, B.C. 7, 251

CN-transitions : J=1 rotational level of ground state of $B^2\Sigma$ multiple
J=0 rotational level

Temperature $\sim 2.3\,K$

Bondi-Gold-Hoyle argument of the mid 1950s.

Total matter density $= 3 \times 10^{-31}\,g\,cm^{-3}$

Helium present $= \frac{1}{4} \times 3 \times 10^{-31}\,g\,cm^{-3}$

$= 7.5 \times 10^{-32}\,g\,cm^{-3} \equiv \rho_{He}$

1 gm of H $\rightarrow$ He makes radiation 6×10^{18} erg.

$\Rightarrow$ Radiation density produced in making ρ_{He}.

$= 7.5 \times 10^{-32} \times 6 \times 10^{18} = 4.5 \times 10^{-13}\,erg\,cm^{-3}$.

[Observed starlight energy density $\lesssim 10^{-14}\,erg\,cm^{-3}$]

Gold argued that if this were thermalized it would produce a temperature of 2.78K.

Bondi & Gold over-ruled in favour of redshifted IR-background. [They did not recall McKellar's work]

Primordial interpretation rests on cooling of the universe to a temperature when most electrons get trapped by the Coulomb attraction of protons to form neutral H-atoms.

Saha's ionization equation (~1920)

$$e, \; p, \quad e+p \; (\equiv H).$$

$$N_e = N_p, \quad N_H$$

In thermodynamic equilibrium at temperature T,

$$\frac{N_e^2}{N_H} = \left(\frac{m_e kT}{2\pi\hbar^2}\right)^{3/2} \exp\left(-\frac{B}{kT}\right)$$

$B = 13.59 \text{ eV}$

[Compare with D-equation or ^4He eqn.
$B_D \cong 2.22 \text{ MeV}$, $B_{He} \cong 28.29 \text{ MeV}$.]

As $T \to 3000K$ most free electrons disappear & radiation travels freely.

$\to$ Last scattering surface at $T \sim 3000 K$

$$1+z = \frac{S_0}{S} = \frac{T}{T_0} = \frac{3000}{2.7} \gtrsim 1100.$$

At or beyond $z \approx 1100$ the universe is opaque. Astronomers cannot observe it!

IX

Not yet understood:

a) The photon-baryon ratio

$$\frac{N_\gamma}{N_B} = 3.33 \times 10^{7} \left(\Omega_B h_0^2\right)^{-1} \left(\frac{T_0}{2.7}\right)^3$$

b) The coefficient in $\rho_B = \eta\, T_9^3$ in primordial nucleosynthesis era.

These have to be 'put in by hand'!

Ref: guesses of T_0 by Alpher & Herman and by Gamow.

c) Relationship to non-baryonic matter.

<u>However</u>: Given the existence of the last scattering surface one can use its structure to a) check on ideas relating to the very early universe ($z \gg 1100$) and to b) provide initial conditions for evolution of large scale structure and the nature of Friedmann model

Discuss (a) first, then (b).

The 'very early' universe

Cosmology & particle physics

$$\frac{\dot{S}^2}{S^2} = \frac{8\pi G}{3}\rho$$

$$\rho c^2 = \tfrac{1}{2} g a T^4$$

$$g = g_b + \tfrac{7}{8} g_f$$

$$S \propto t^{1/2} \longrightarrow t = \left(\frac{3c^2}{16\pi G a}\right)^{1/2} g^{-1/2} T^{-2}$$

In thermodynamic equilibrium typical particle energy is kT. Express kT in MeV or GeV.

$$t_{second} = 2.4\, g^{-1/2} T_{MeV}^{-2} = 2.4\times 10^{-6} g^{-1/2} T_{GeV}^{-2}$$

Grand unification of electromagnetic + weak + strong interactions is expected at $\sim 10^{15}$ GeV

$$T_{GeV} \sim 10^{15} \Rightarrow t_{second} \approx 2.4\times 10^{-6} \times 10^{-30} g^{-1/2}$$

$$g \sim 100 \Rightarrow t \sim 10^{-37} - 10^{-36} \text{ second !}$$

Particle accelerators go up to 10^3 GeV.
So there is a gap of 10^{12} between theory & experiment

"The Early Universe is a poor man's high energy accelerator".

X

A conceptual problem

Padmanabhan & Vasanthi in the 1980s pointed out that numbers of particles are not large enough to justify flat spacetime thermodynamics & statistical mechanics.

For principle of equivalence to apply one needs a 'locally flat region "Q" at epoch t.

Size of Q $= L$ say.

$L \ll$ local radius of curvature $\sim R^{-1/2} \sim ct$

Write $L = \alpha ct$, $\alpha \ll 1$.

No. of particles in Q $\sim 2.4 \times \frac{g}{\pi^2} \times \left(\frac{kT}{c\hbar}\right)^3 \times \frac{4\pi}{3} L^3$

$$\mathcal{N} \sim g\left(\frac{kT}{c\hbar}\right)^3 L^3 = g\left(\frac{kT}{c\hbar}\right)^3 \alpha^3 c^3 t^3$$

Write 'Planck Temperature' as $T_P = \hbar / k t_P$

$t_P =$ Planck time $= \sqrt{\frac{G\hbar}{c^5}} \sim 10^{-43}$ s. Use T–t relation

$$\mathcal{N} \sim \frac{\alpha^3}{30\sqrt{g}} \left(\frac{T_P}{T}\right)^3$$

$\alpha \ll 1 \Rightarrow \mathcal{N}$ is not large unless $T \ll T_P$.

At GUT epoch $T \approx 10^{15}$ GeV, $T_P \approx 10^{19}$ GeV, $g \approx 100$

$$\mathcal{N} = \left(\frac{\alpha}{10^{-3}}\right)^3 \times 3$$

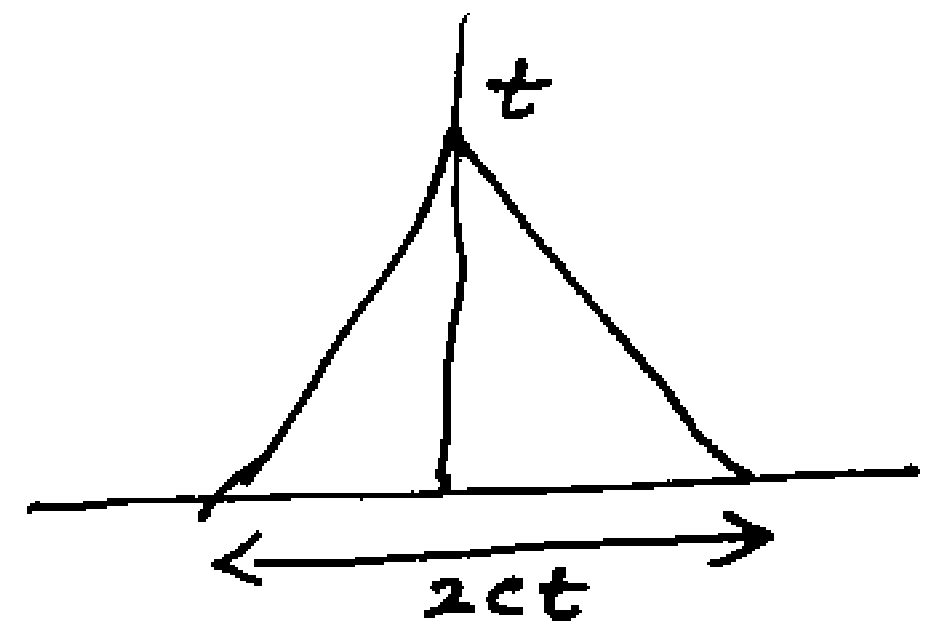
For $\alpha \sim 10^{-6}$, $\mathcal{N} \lesssim 10^{-9}$, for $\alpha = 10^{-3}$, $\mathcal{N} = O(1)$.

We cannot apply flat spacetime stat. mech. laws here!

I

Other conceptual problems that have been recognized.

1. The horizon problem



Particle horizon at time t

Homogeneity at t cannot causally extend beyond a region of diameter 2ct.

Let temperature at time t is T.

Present day temperature is $T_0 \sim 2.7K \approx 2.3 \times 10^{-13}$ GeV.

$$T \propto \frac{1}{S}$$

$\Rightarrow$ Expansion by a factor $\frac{T}{T_0}$

At GUT epoch $T \sim 10^{15}$ GeV.

$$\frac{T}{T_0} \cong 4 \times 10^{27}$$

The GUT horizon has expanded to a size $2ct \times 4 \times 10^{27}$ in diameter.

$t \sim 10^{-37}s \Rightarrow 2ct \sim 2 \times 3 \times 10^{10} \times 10^{-37}$ cm $\sim 6 \times 10^{-27}$ cm

Present size $= 4 \times 10^{27} \times 6 \times 10^{-27} \approx 24$ cm!

Too small!

8

2. The Flatness problem

When can we omit the 'k' term in

$$\frac{\dot{S}^2 + kc^2}{S^2} = \frac{8\pi G\rho}{3} \quad ?$$

$\rho = \rho_c \Omega$

$$\frac{kc^2}{S^2} = (\Omega - 1)\frac{\dot{S}^2}{S^2}$$

If we use $S \propto t^{1/2}$ $\qquad \frac{kc^2}{S^2} = \frac{\Omega - 1}{4t^2}$.

At present $\quad \frac{kc^2}{S_0^2} = (\Omega_0 - 1) H_0^2$

$$\Rightarrow \quad \Omega - 1 = 4 H_0^2 t^2 \cdot \frac{T^2}{T_0^2} (\Omega_0 - 1) \qquad \text{since } S \propto 1/T$$

$$\simeq 4.3 h_0^2 \times 10^{-53} (\Omega_0 - 1) \qquad H_0 = 100 h_0 \text{ km s}^{-1}\text{Mpc}^{-1}.$$

at GUT epoch.

Fine-tuning of exceptionally high order!

3. Monopole problem

Breakdown of symmetry from a larger group like $SU(5)$ to $SU(3) \times SU(2)_L \times U(1)$ leaves monopoles as indestructible particles. These are magnetic monopoles not allowed in Maxwell's theory. Monopole mass $\simeq 10^{16}$ GeV/c^2.

Assume only one monopole per GUT horizon sphere of radius $2ct$.

$$\Rightarrow \text{monopole density of mass} = \frac{10^{16}\ \text{GeV}/c^2}{\frac{4\pi}{3}(2ct)^3}.$$

Today this decreases to $\left(\frac{S}{S_0}\right)^3 = \left(\frac{T_0}{T}\right)^3$ of the primordial value. Today's monopole mass density is

$$\rho_M \cong 1.5 \times 10^{-13} \left(\frac{T_0}{2.7K}\right)^3 g\ cm^{-3}$$

$$\cong 10^{16} \times \text{ closure density!}$$

How can we get rid of relic monopoles?

4. Entropy problem

Entropy in a given comoving volume stays constant (Adiabatic expansion!)

The present photonic entropy in the observable univer[se] of radius $R \approx h_0^{-1} \times 10^{28}$ cm is

$$\Sigma = \frac{4\pi}{3k} a T_0^3 R^3 \approx h_0^{-3} \times 4.4 \times 10^{87} \left(\frac{T_0}{2.7K}\right)^3.$$

Why such a large value?

Is Σ = constant violated at some stage?

Recall the photon/baryon ratio of $10^8 - 10^{10}$ which is not understood yet!

X

'Inflation' proposed as a solution to 1–4.

Guth 1981 Phys. Rev. D23, 347

Kazanas 1980 Ap. J. 241, L59

Sato 1981 MNRAS 195, 467

Gut phase transition produces a 'false' vacuum which then converts to 'true' vacuum by releasing energy that makes the universe expand rapidly.

$t < t_1 \qquad S \propto t^{1/2}$

$t_1 < t < t_2 \qquad S \propto \exp(t/\tau) \longrightarrow$ Inflation

$t > t_2 \qquad S \propto t^{1/2}$

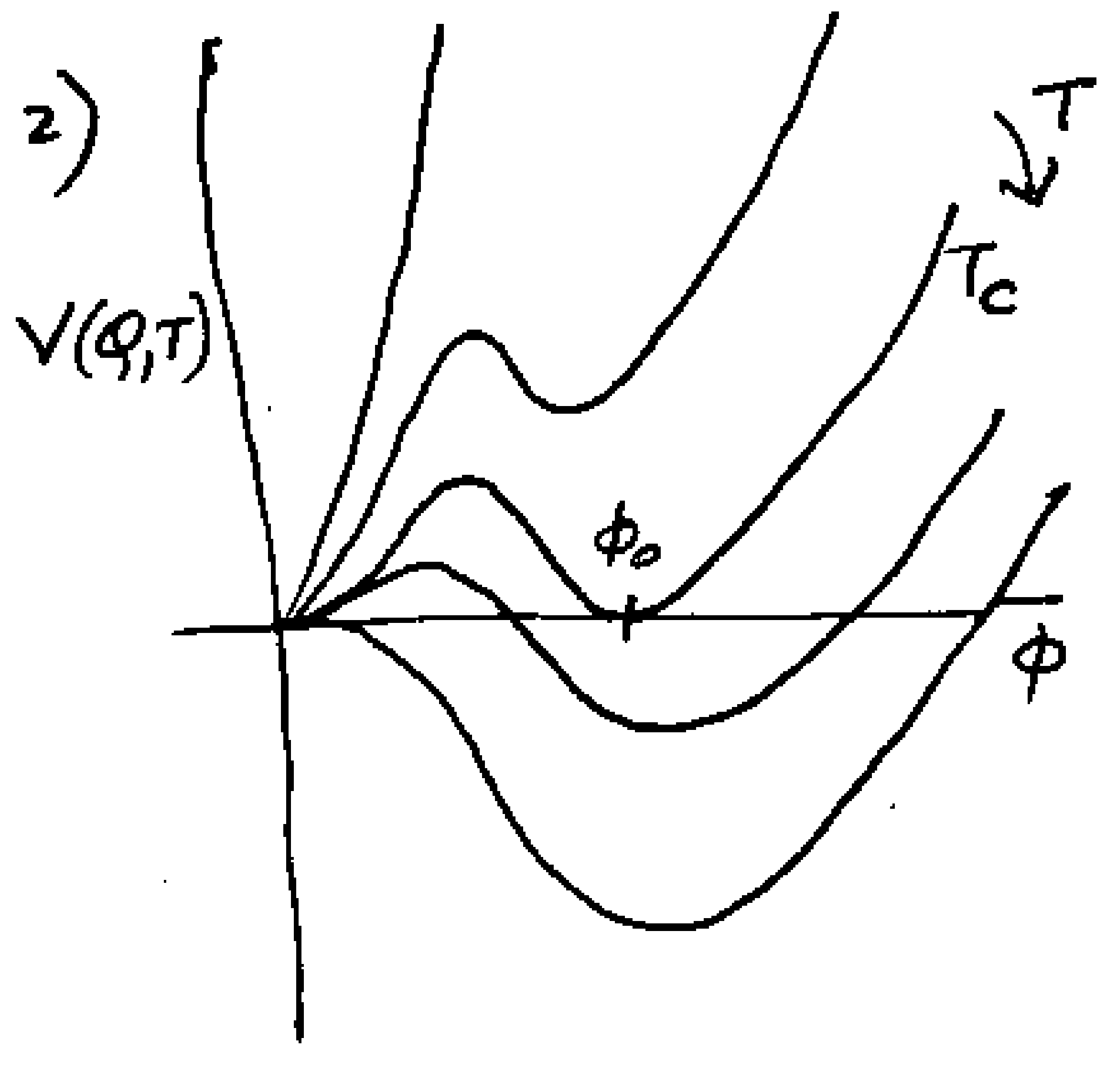
Phase transition is brought about by a Higgs field ϕ with a potential

$$V(\phi, T) = V_{eff}(\phi) + \frac{18\,T^4}{\pi^2}\int_0^\infty x^2 \ln\left\{1 - \exp\left[-\left(x^2 + a\frac{\phi^2}{T^2}\right)^{1/2}\right]\right\}dx$$

$$V_{eff}(\phi) = \alpha\phi^2 - \beta\phi^4 + \gamma\phi^4 \ln(\phi/\sigma^2)$$

For $T < T_c$ the true vacuum is at $\phi > 0$.

For $T > T_c$ the true vacuum is at $\phi = 0$.



$$\frac{\dot{S}^2 + kc^2}{S^2} = \frac{8\pi G}{3c^2}(\epsilon_0 + \epsilon_r)$$

ϵ_r = radiative energy (γ and relativistic particles)

ϵ_0 = extra vacuum energy

$\epsilon_r \propto 1/S^4$ $\quad \epsilon_0 \approx$ constant $>> \epsilon_r$

$$S \propto e^{(t/\tau)} \qquad \tau = \left(\frac{3c^2}{8\pi G \epsilon_0}\right)^{1/2}$$

Transition by tunneling from $\phi = 0$ to $\phi = \phi_0 > 0$.

Time taken for it $\cong \tau_0$ (say). After phase transition is over the region expands like a Friedmann Bubble.

S increased by $\exp(\tau_0/\tau) \equiv Z$

For $\tau_0 \approx 67\tau$ $\quad Z = 10^{29}$.

Horizon expanded suddenly by a large factor!

The kc^2/S^2 term becomes negligible

Monopoles are expanded away by factor Z^{-3}.

But the entropy problem remained!

If bubbles collide they will heat & dump the excess entropy into the universe.

But the bubbles move apart exponentially & never collide!

I "New" Inflation

Coleman - Weinberg potential

$$V_{eff}(\phi, T) = \frac{25}{16}\alpha^2\left[\phi^4 \ln\left(\frac{\phi^2}{\sigma^2}\right) + \frac{1}{2}(\sigma^4 - \phi^4)\right] + \frac{18}{\pi^2}T^4\int_0^\infty \ln\left\{1 - \exp\left[-\left(x^2 + \frac{5}{12}\phi^2\frac{g^2}{T^2}\right)^{1/2}\right]\right\}dx$$

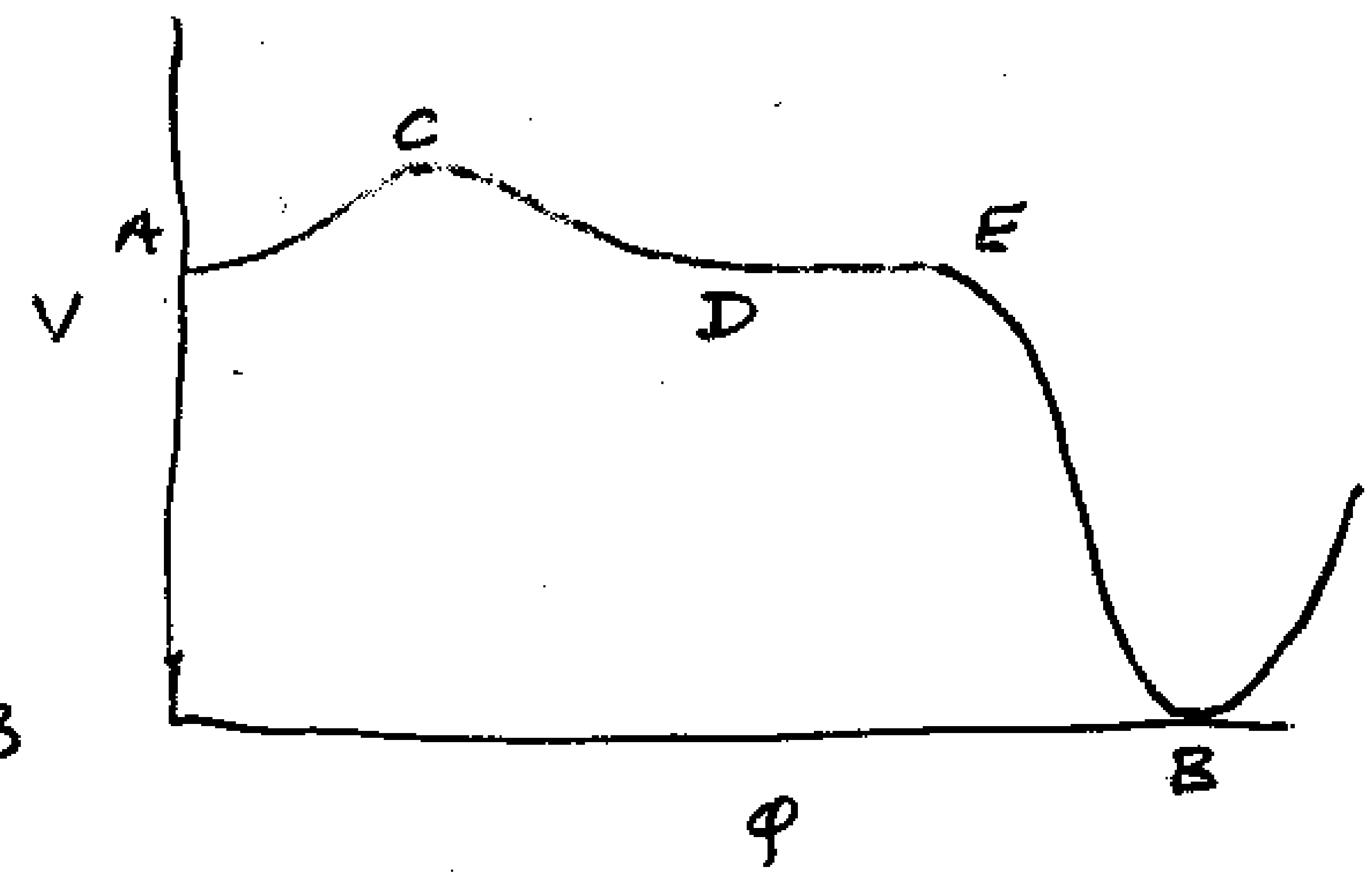
A: False vacuum

B: True vacuum

C: Temp - dependant bump

D: Slow roll down

E: Drop to lower state B



All the parameters have to be finely tuned, however!

Hubble constant for inflation $\cong 2 \times 10^{10}$ GeV $(\equiv H)$

Time of roll-over $= 190 H_\phi^{-1}$

Z = exponential expansion $\approx \exp 190 \approx 10^{50}$

Time of oscillation before settling down at $\phi \approx \sigma \sim 2\times10^{15}$ GeV

is $\sim \exp(4.8) \times 10^{-4} H_\phi^{-1} \ll H^{-1}$

$\Rightarrow$ Oscillations are damped by decay of ϕ into relativistic particles and radiation.

Reheating raises the temperature of the universe & thus

XI

Some problems with inflation:

1. Small number problem still remains.

2. Fine tuning has not been eliminated.

3. The cosmological problem of λ

$$\frac{\lambda_{\text{observed}}}{\lambda_{\text{GUT}}} \approx 10^{-108}$$

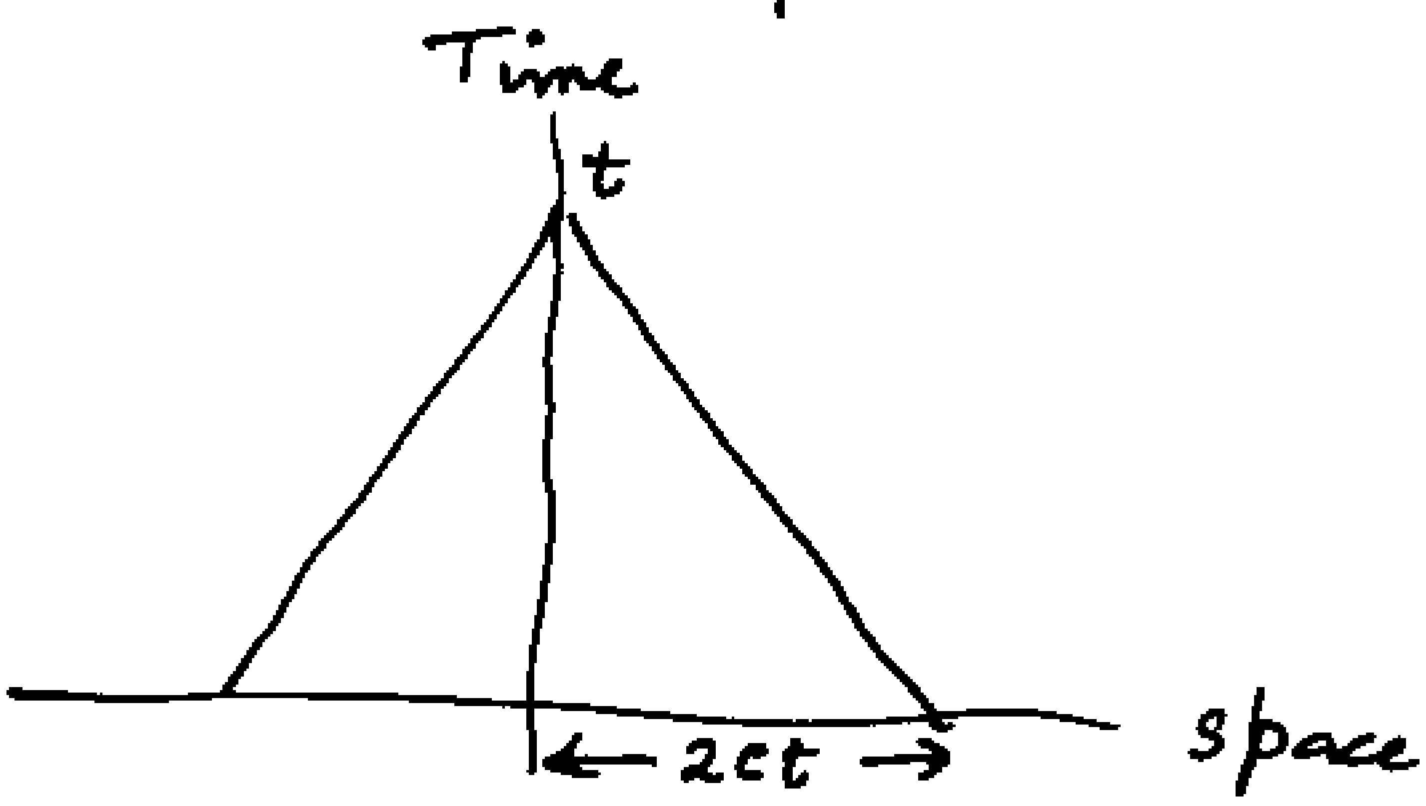
What happened to the large λ? Why only a small fraction left back?

The Quantum Gravity epoch:

$$\mathcal{A}_{\text{Gravity}} = \frac{c^3}{16\pi G} \int_{\mathcal{V}} R \sqrt{-g}\, d^4x$$

Typical component of $R \sim R^0_0 \sim 3/4c^2t^2$.

At ~~time~~ t take $\mathcal{V}$ bounded by particle horizon sphere of ~~radius~~ $2ct$ and time span ct.



XI

$$A_{gravity} = \frac{c^3}{16\pi G}\int_0^t (2ct_1)^3 \times \frac{3}{4c^2 t_1^2}\, c\, dt_1$$

$$= \frac{c^5}{4G} t^2$$

For quantum gravity $A_g \sim \hbar$

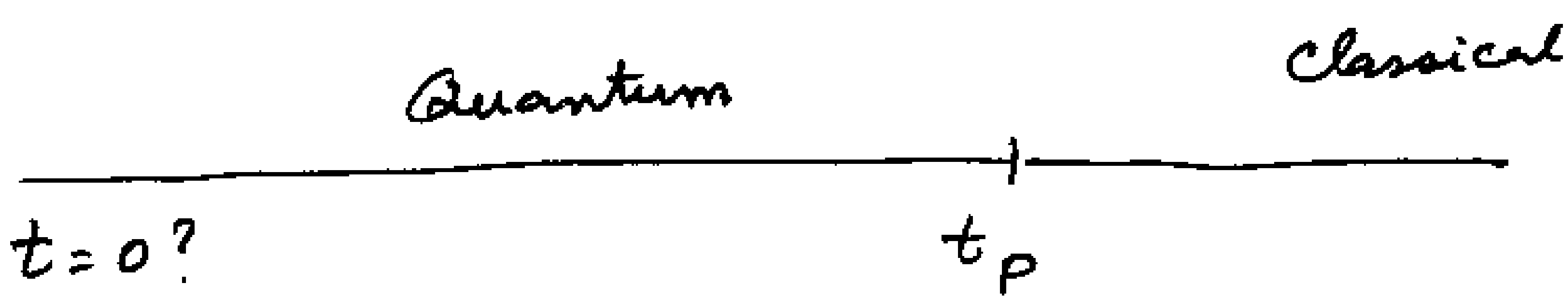
$$\frac{c^5}{4G} t^2 \sim \hbar$$

$$t \sim 2\sqrt{\frac{G\hbar}{c^5}} \rightarrow 2t_p$$

$$t_p \equiv \sqrt{\frac{G\hbar}{c^5}} \quad \text{Planck time}$$

Planck energy $E_p = \dfrac{\hbar}{t_p} \sim 10^{19}\,\text{GeV}$

The universe was <u>not</u> determined by classical GR.



Can we determine classical $\leftrightarrow$ quantum relationship?

Yes! in case of conformal fluctuations.

Formation of large scale structure

See book : Large Scale Structure of the Universe by T. Padmanabhan
Cambridge University Press 1993

The work on LSS is the main 'industry' in theoretical cosmology today.

Discuss here only some highlights.

'Standard' scenario has the following major epochs:

1. The Planck Epoch
2. The GUTs / Inflation Epoch
3. The Recombination Epoch

$\uparrow\downarrow$

4. The epoch when the universe became matter dominated
5. The epoch of $z \sim 5-6$ when galaxies began to form
6. Hierarchy of structures form

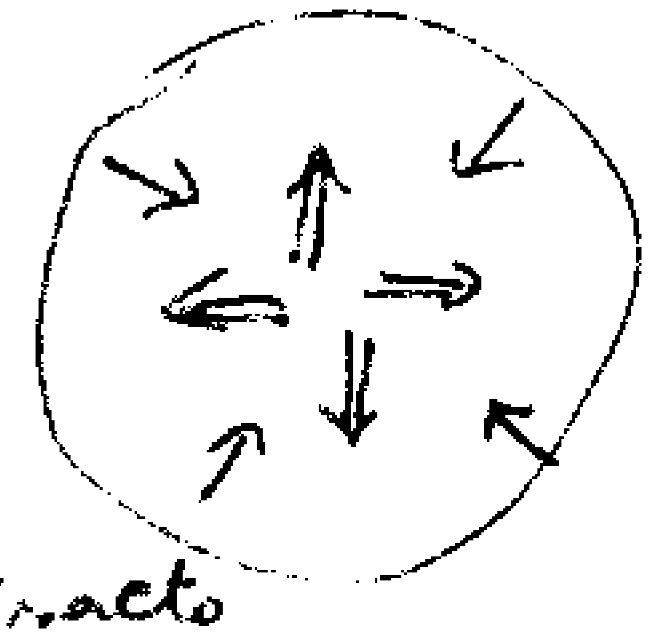
Things that may influence : (1) Dark Matter $\rightarrow$ mainly nonbaryonic (2) Dark energy $\leftrightarrow \lambda$ (3) Biasing (4) Impact on radiation background (5) How much random motion is developed.

XII

<u>Jeans mass</u>: Work of James Jeans in 1902: Phil Trans, R.S. <u>199</u> A, 49

→ Gravity

⇒ Pressures



Whether a given mass contracts or oscillates depends on whether → or ⇒ dominates.

n = number density of particles

m = mass per particle

$\rho = nm$ = density

c_s = sound velocity in the medium

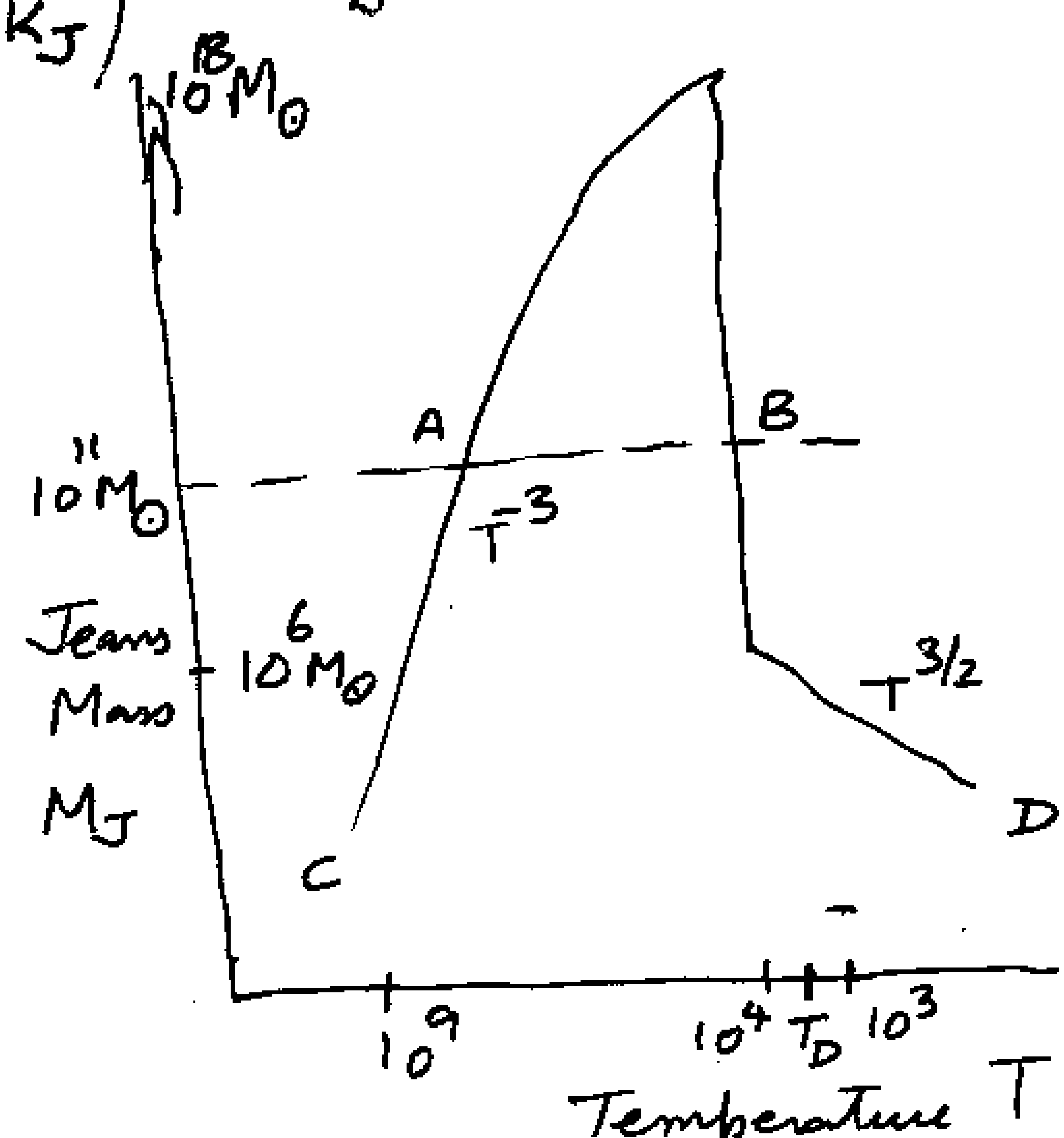
Jeans wave number $\quad K_J = \left(\dfrac{4\pi G\rho}{c_s^2}\right)^{1/2}$

$$\text{Jeans mass} = \frac{4\pi n m}{3}\left(\frac{2\pi}{K_J}\right)^3 \equiv M_J$$

Contraction if $M > M_J$

Oscillation if $M < M_J$

A galaxy size mass grows between CA & BD but not between AB.



$$\delta = \frac{\delta\rho}{\rho}$$

Growth equation tells us how δ grows from B to D.

From decoupling epoch to present δ may grow at most by a factor $\sim 10^3$

If $\frac{\delta\rho}{\rho}$ today is at least 1, it was $\sim 10^{-3}$ at decoupling.

At or before decoupling radiation & matter had same order of fluctuation.

$$\Rightarrow \quad \left.\frac{\delta\rho}{\rho}\right|_{Rad} = \left.\frac{\delta\rho}{\rho}\right|_{Matter} = 10^{-3}$$

$$\rho_{Rad} \propto T^3 \qquad \left.\frac{\delta\rho}{\rho}\right|_{Rad} = 3\,\frac{\delta T}{T}$$

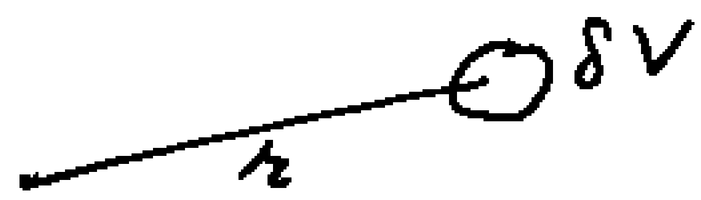
$$\Rightarrow \quad \frac{\delta T}{T} \sim 3\times 10^{-4}$$

No such fluctuations were found.

$\Rightarrow$ To save the theory, most of the matter has to be non-baryonic, i.e., uncoupled to radiation.

Then $\left.\frac{\delta\rho}{\rho}\right|_{Rad} << \left.\frac{\delta\rho}{\rho}\right|_{matter}$.

Scale-invariant spectrum

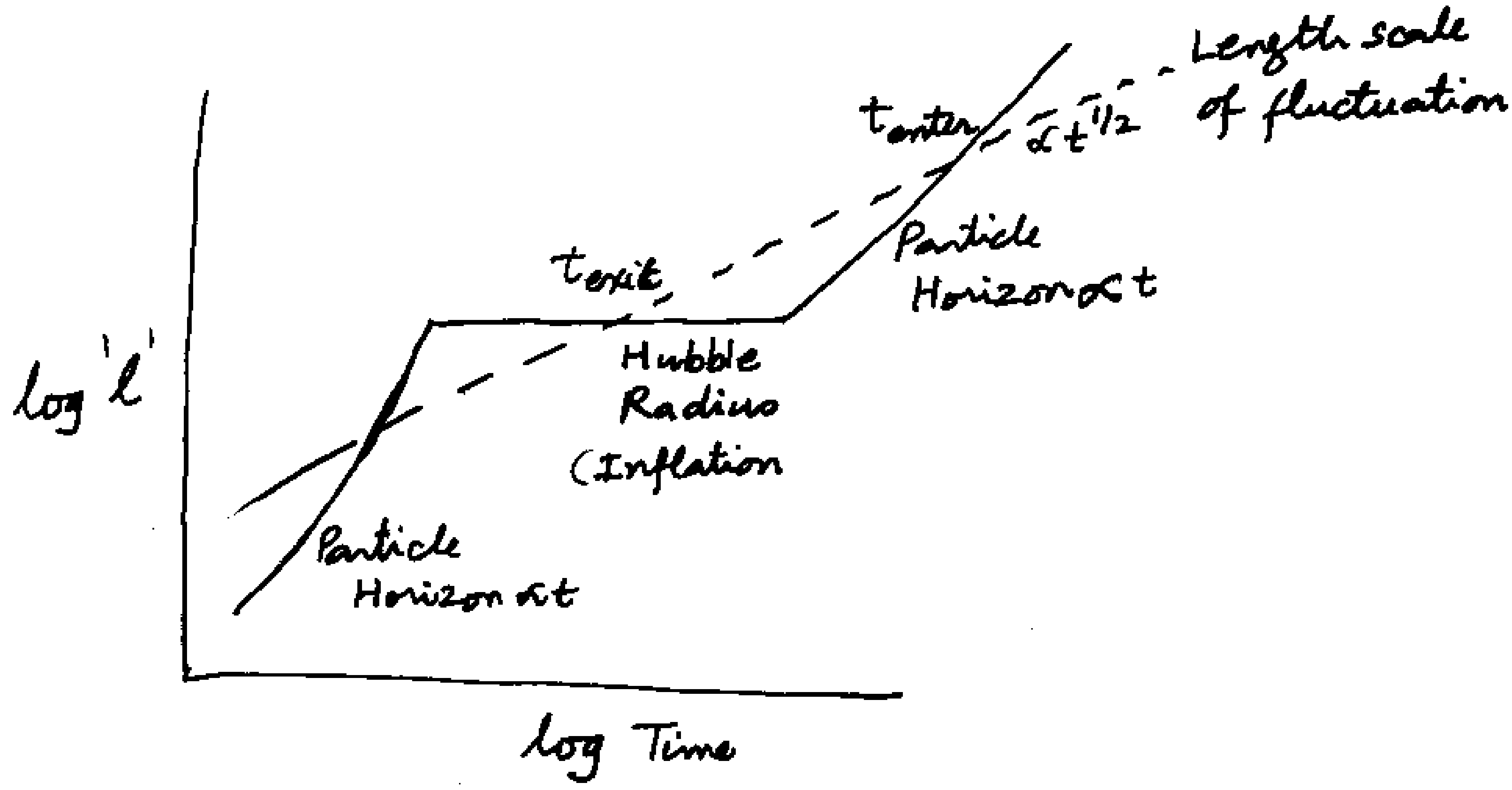


$$\delta(r) = \bar{n}\left[1+\xi(r)\right]\delta V$$

$$\xi(r) = \left(\frac{r}{r_0}\right)^{-\gamma} \qquad \gamma = 1.8$$

This behaviour is found for galaxies in clusters and for clusters in superclusters. Also for superclusters?

Why inflation helps?



One can fold in inflationary model to study growth of fluctuations of different length scale. One can obtain a scale invariant spectrum by adjusting parameters of the model. However $\delta \gg 1$ instead of $\delta < 10^{-4}$ unless some fine-tuning is done!

Nature of dark matter

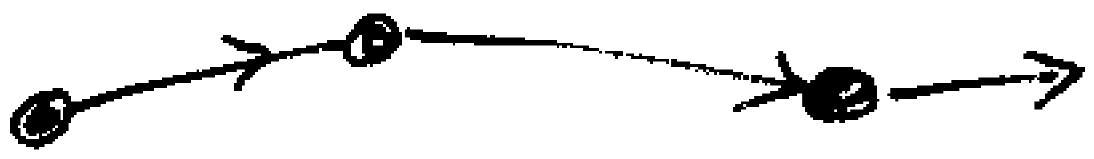
'Hot' if moving fast when decoupled, e.g. neutrinos with non-zero mass. $\rightarrow$ superclusters form first

'Cold' if moving slow when decoupled ... axions, photinos sneutrinos etc. $\rightarrow$ masses $\sim 10^6 M_\odot$ form first.

Today HDM is not favoured.
CDM is better but requires dark energy
$\Rightarrow \Lambda CDM$

Zel'dovich approximation & N-body simulations

Attempt to deal with non-linear regime.



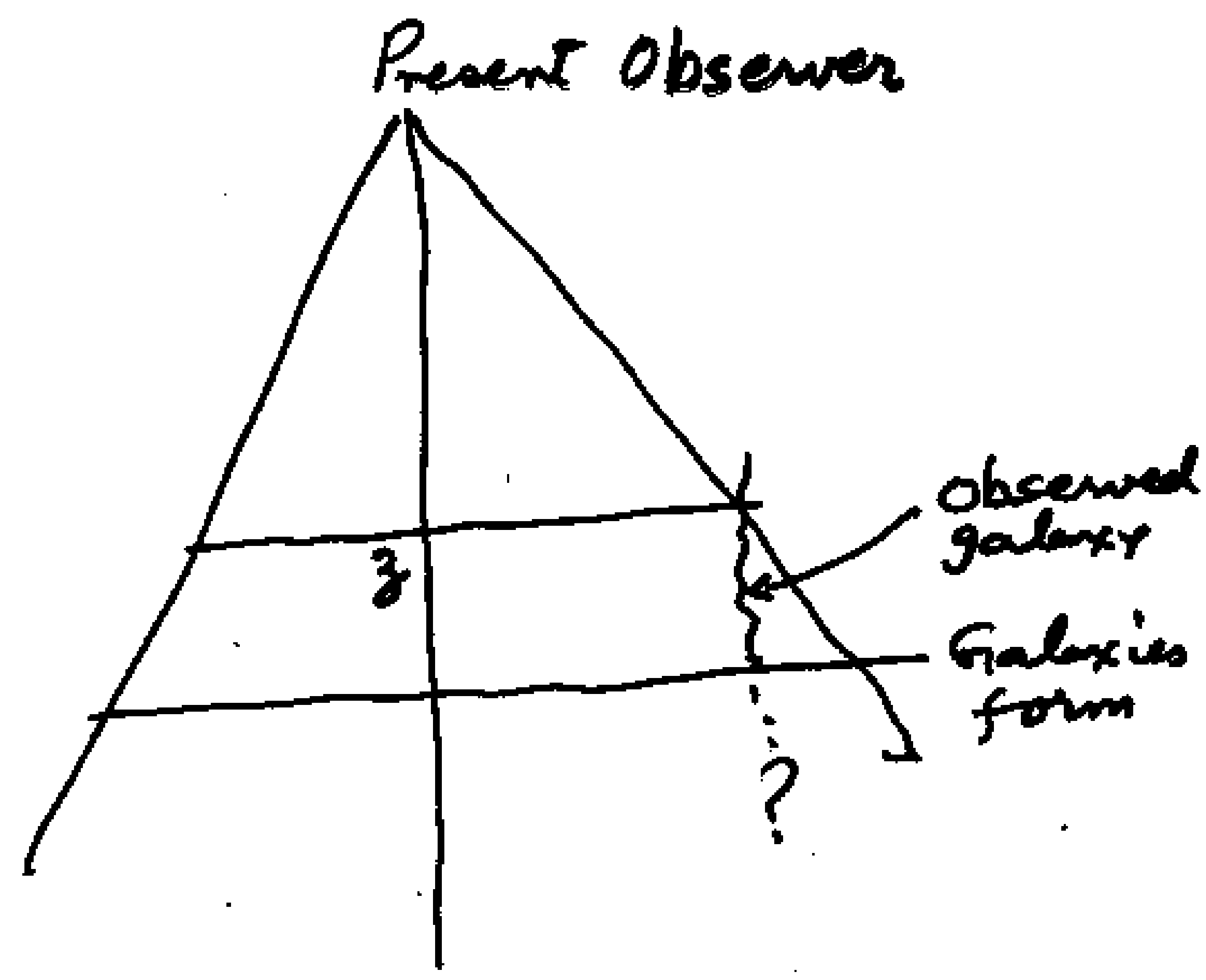
fluid element moves under the existing gravitational force of the matter.

Pancakes develop $\longrightarrow$ $\delta\rho/\rho$ diverges on 'planes'.

N-body simulations are meant to tackle the non-linear regime.

Age-problem: Fully formed galaxies must be developed early enough! Also, the ages of galaxies seen at high redshifts must be low enough!

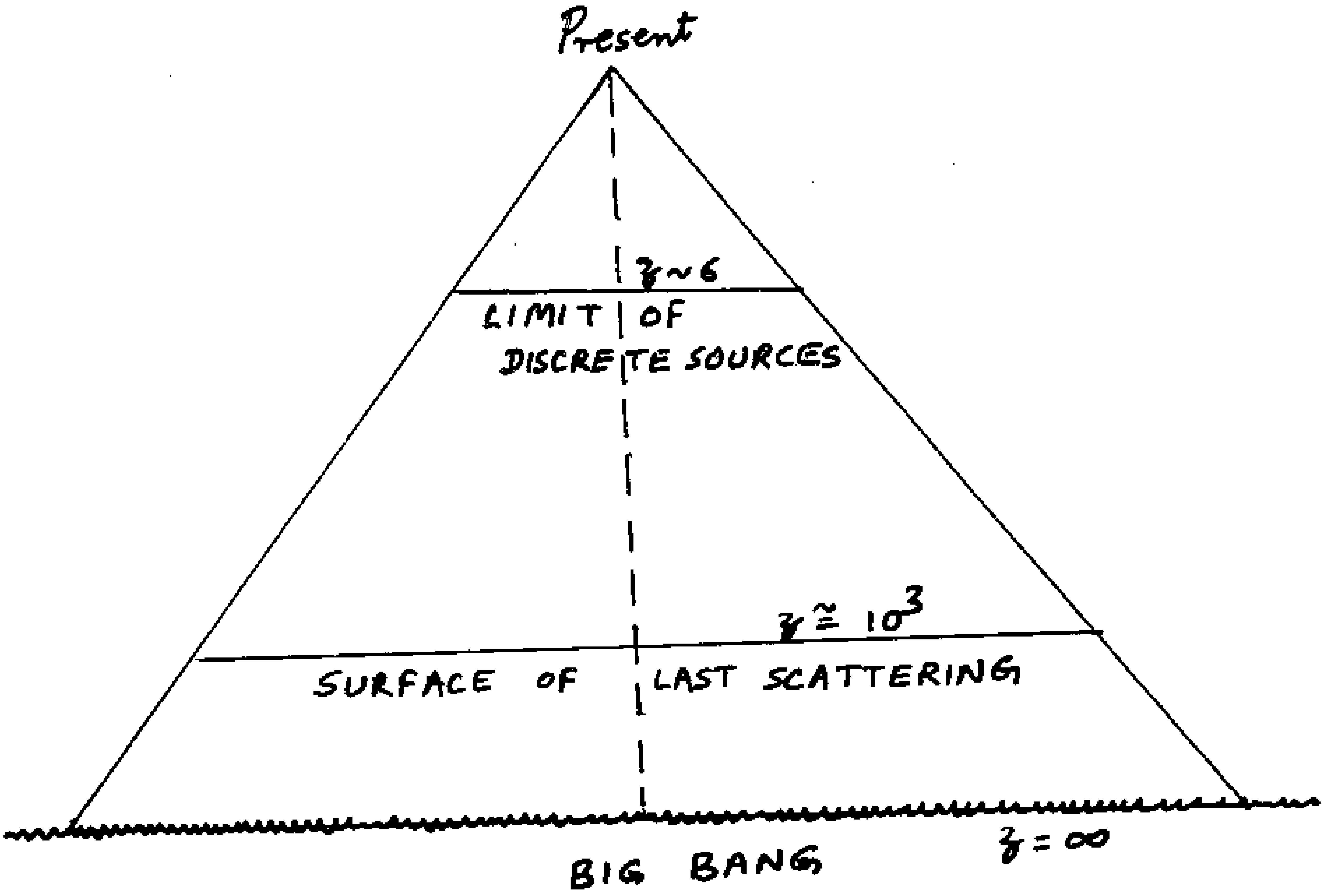
'λ' might help.... without λ the models run into problem.



Observational Tests of Cosmological models

→ of two kinds

(I) Direct ... observations of distant parts of the universe



Advantage : Sources are directly observed upto $z\sim 6$

Disadvantage : Physical evolution may confuse the issue

(II) Indirect ... local observations are used to infer the distant past

e.g. CMBR ... $z\sim 10^3$

Nucleosynthesis of light nuclei ... $z\sim 10^8-10^9$

Advantage : information about very early universe.

Disadvantage : large extrapolations + speculative physics.

e.g. inflation!

(I) Direct observations

A. Value of H_0: Many methods are available, e.g.

standard candle ... cepheid variables
brightest star
planetary nebulae
H II regions (size)
Type Ia supernova ... brightest phase
Type II supernova ... rate of expansion of photosphere

Tully-Fisher relation ... Correlation between luminosity of a spiral galaxy & its 21 cm line width.

For ellipticals:
3-D relation → $\log R$, $\log \sigma$, $\log I$ lie on a 'fundamental plane'

R ~ effective radius of elliptical
I ~ average surface brightness
σ ~ velocity dispersion

} Davis & Djorgovski

Faber-Jackson relation L & σ in central region only.
Gravitational lensing time delay

Despite several methods, the measurements of H_0 are still uncertain

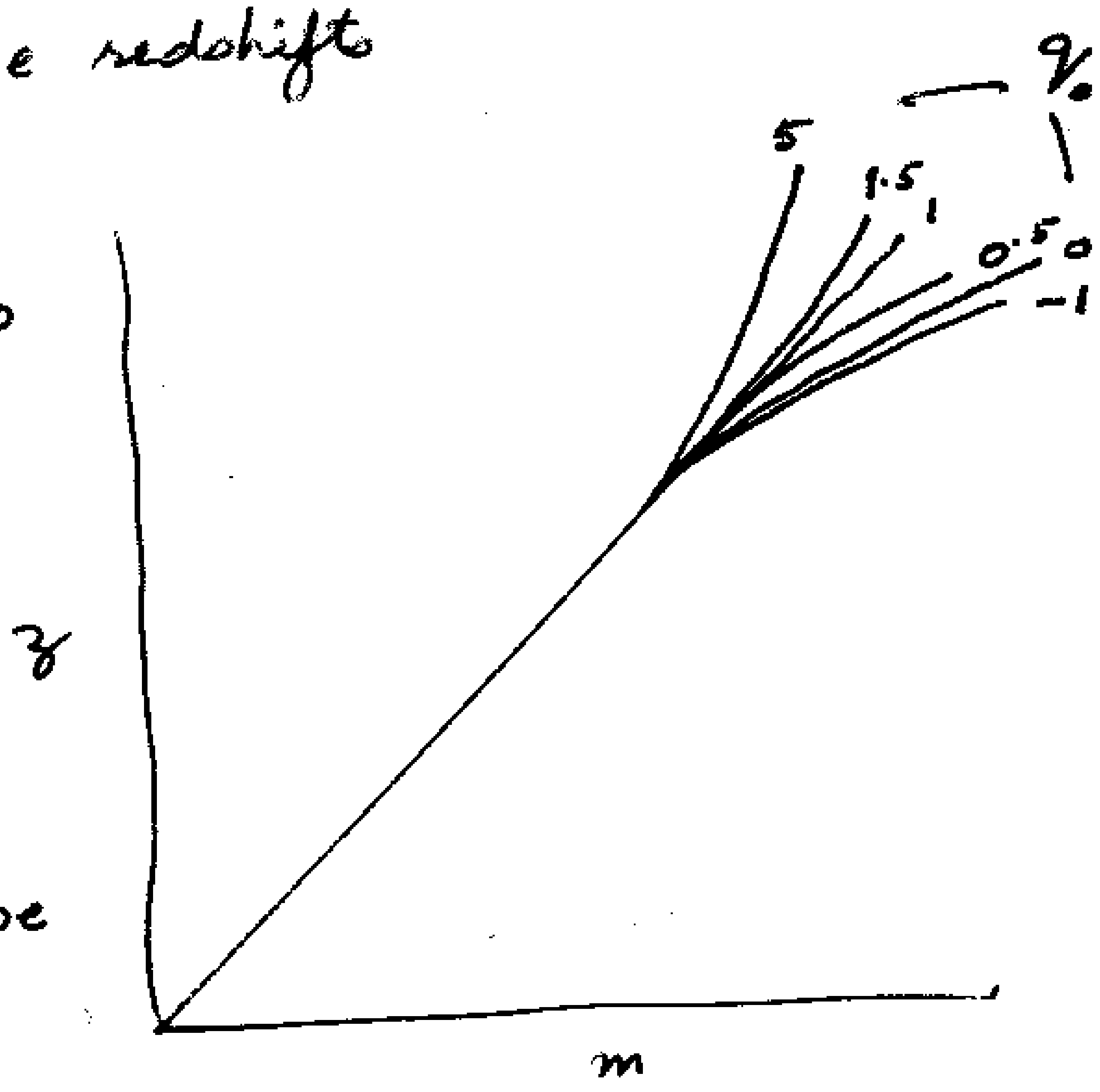
Favoured value ~ $H_0 = 70 \ km\,s^{-1}\,Mpc^{-1}$
Sandage, Tammann ~ $H_0 = 50 \ km\,s^{-1}\,Mpc^{-1}$
Gravitational lensing ~ $H_0 \simeq 40 \ km\,s^{-1}\,Mpc^{-1}$

B. Hubble relation to large redshifts

The values in the 1960-80 period preferred

q_0 in the range $[0 \leftrightarrow 1]$

i.e., a decelerating universe



Only the steady state theory gave an accelerating universe : $q_0 = -1$

In the post-1998 period the use of Type Ia supernovae as standard candles has suggested $q_0 < 0$, i.e. an accelerating universe.

Simple Friedmann models with $\lambda = 0$ do not fit.

Use $\lambda > 0$ models.

Best fit gives $\Omega_0 + \Omega_\Lambda > 1$ (~ 1.2)

However, if one accepts inflation, $\Omega_0 + \Omega_\Lambda = 1$. The best such case has $\Omega_0 = 0.28$, $\Omega_\Lambda = 0.72$.

That is, 'dark energy' dominates the universe.

$$\Omega_0 = \underset{0.04}{\underset{\downarrow}{\Omega_B}} + \underset{0.24}{\underset{\downarrow}{\Omega_{NB}}}$$

C. Number counts

(Extending Hubble's programme)

Radio sources 1950-65

Ryle Cambridge

Mills Australia

Bolton Australia / Caltech

Results inconclusive because:

a) radio sources do not directly give their z

b) there are evolutionary effects

c) statistical effects of local inhomogeneities

Galaxy counts (> 1979) : computer counting of images.

Expected: $\log N = 0.6\,m + \text{constant}$

Normally fit is good at bright end (low m).

At large m, the discrepancy is more at blue band

ie, there seems to be an excess of blue galaxies.

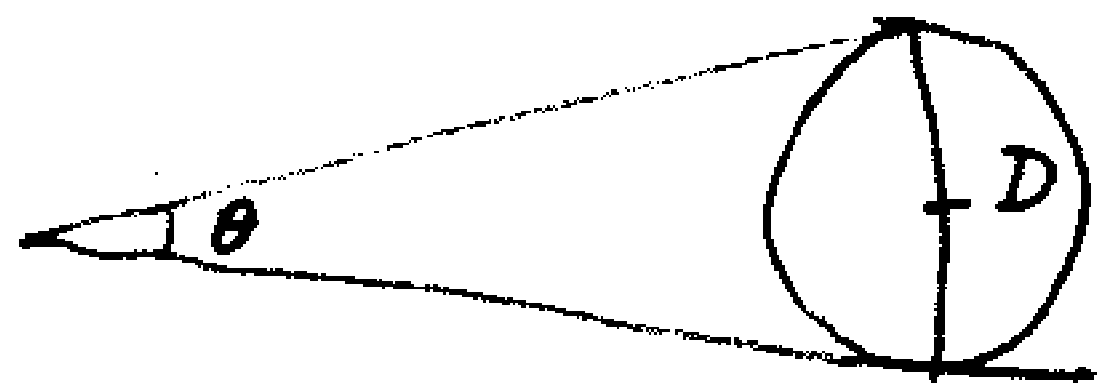
→ starburst galaxies?

→ mergers?

→ dwarf galaxies that fade away at present epoch?

i.e., we learn more about galaxies than about cosmology!

D. Angular diameter - z relation



Angular size $= \theta$, Linear size D

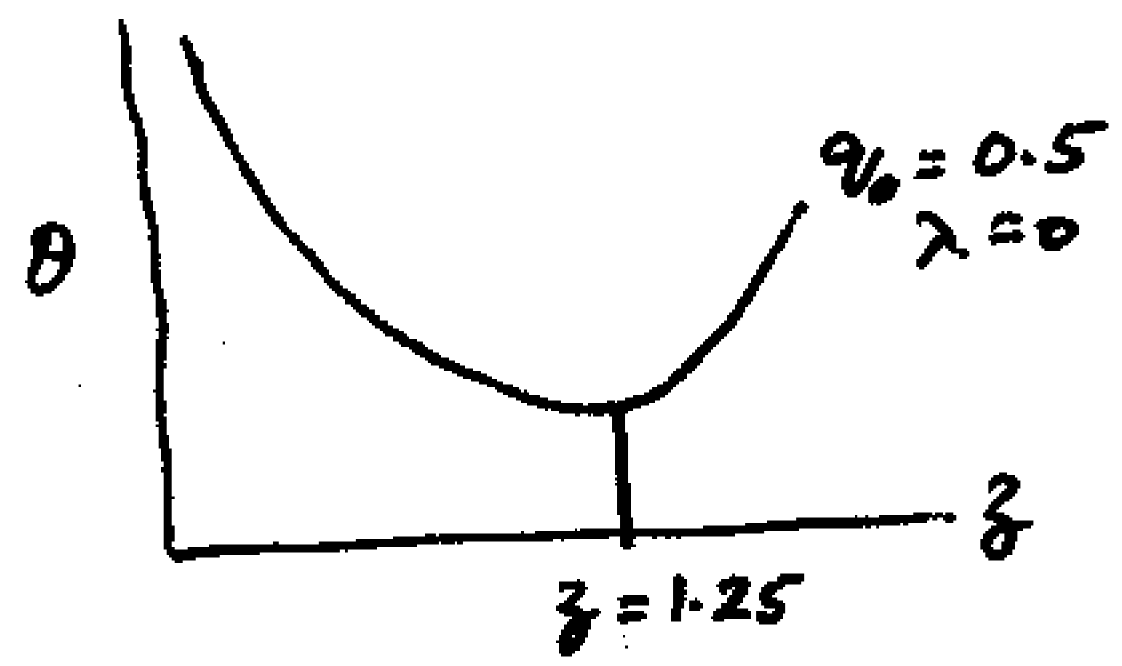
$$ds^2 = c^2 dt^2 - S^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta \, d\phi^2\right)\right]$$

Choose $t = \text{const}$, $r = \text{const}$. $\phi = \text{const}$

$$D = S(t)\, r\, \theta \qquad \theta = \frac{D}{r S(t)}.$$

But $\quad \dfrac{S(t_0)}{S(t)} = 1+z \quad \Rightarrow \quad \theta = \dfrac{D(1+z)}{r S(t_0)} \propto \dfrac{1+z}{r}.$

Thus at large z, θ should start increasing... [Fred Hoyle 1959]



Is this effect observed?

Results for radio galaxies are confused by

1. Projection effects
2. Intrinsic variation of D
3. Evolution of D.

$\Rightarrow$ Attempts to reduce 2 & 3 for quasar interiors.

Result still inconclusive!

(II) Local (Indirect) Observations

A. Anisotropy of large velocity fields

Rubin - Ford Effect

The 'Great Attractor'

Is Hubble constant different for local & distant galaxies?

B. Evidence for matter density

Visible ... $\rho \sim 4 \times 10^{-31} \, g \, cm^{-3}$

$$\rho_{critical} \sim 2 \times 10^{-29} h_0^2 \, g \, cm^{-3}$$

$$\Omega_V = \frac{4 \times 10^{-31}}{2 \times 10^{-29} h_0^2} \sim .02 h_0^{-2}$$

$$\Omega_V h_0^2 \sim 0.02$$

Dark matter density ρ_D is to be added to this.

From clusters, using Virial theorem one arrives at

$$\Omega_D \lesssim \frac{0.15 h_0^{-1/2}}{1 + 0.55 h_0^{3/2}}$$

Deuterium abundance limit $\Omega_B h_0^2 \lesssim 0.024$

Thus most of the dark matter has to be non-baryonic if BBN is right.

So far there is no direct evidence for non-baryonic matter. But $\Omega_B + \Omega_{NB} + \Omega_\Lambda = 1$ requires $\Omega_{NB} \sim .24$ for $h_0 \sim 0.7$

C. Age of the universe

Nuclear ages $\sim$ 12 Gyr from studies of radioactive decays of matter in stars, supernovae etc.

Stellar ages in globular clusters $\sim$ 14 Gyr ... from main sequence turn off.

Error bars $\sim$ 2-3 Gyr.

$H_0 = 70 \text{ km s}^{-1} \text{Mpc}^{-1}$ $\quad \Omega = 1 \;, \; \lambda = 0$

$$\text{Age} = \frac{2}{3} H_0^{-1} \approx \frac{2}{3} \times 10^{10} \times h_0^{-1} \text{ yrs}$$

$$\approx \frac{2}{3} \times \frac{1}{0.7} \times 10^{10} \text{ yrs} \lesssim 10^{10} \text{ yrs}$$

This is too low! So you need $\lambda > 0$.

Can manage with $\Omega_\Lambda = 0.72$ ... just about!

D. Abundances of light nuclei

By adjusting the coefficient α in

$$\rho = \alpha T^3$$

in the early universe BBN calculations it is possible to understand abundances of ^4He and ^2H. For the latter one needs the requirement that most of the matter in the universe is nonbaryonic!

E. CMBR is considered the 'best' evidence.

a) Spectrum is perfect black body

b) Anisotropies are explained by the various perturbations of the early universe

c) Polarization can be likewise understood.

However, these are all 'consistency' arguments. One can find suitable parameters to fit the various bumps and dips. But the uniqueness of the scenario is not guaranteed.

In conclusion: the present exercise requires (i) large extrapolations of physics (GUTs, Inflation, non-baryonic dark matter) that are not independently checked; (ii) the assumptions of scenarios that cannot be directly observed (Doppler effects of oscillating plasma, Assumed quantum fluctuations, non-analytic methods of structure formation ...) (iii) a vast unobserved territory between $z \cong 6$ and $z \cong 1500$, still open for speculation and (iv) a basic understanding of why the present Temperature of CMBR is 2.7K.