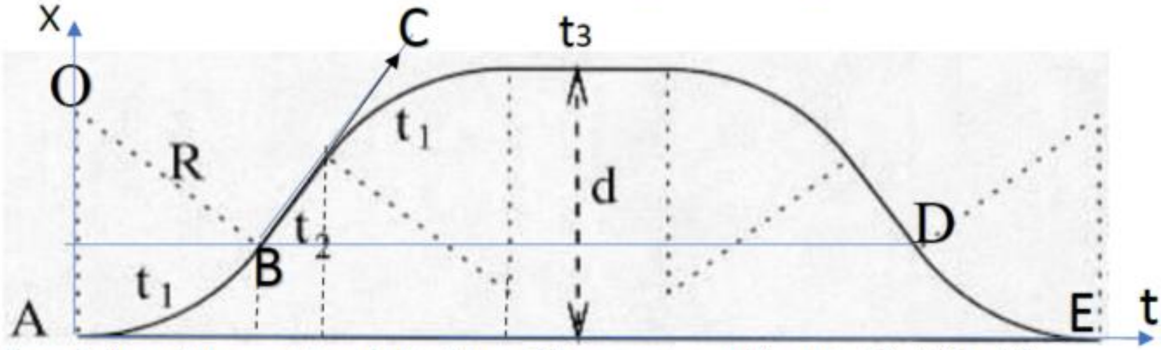
Solution of the twin paradox of Langevin using a Wick rotation.



Description of the problem in Euclidean 2D geometry defined by $L^2 = V^2t^2 + X^2$.

The twins separate at A. The one who remains on Earth follows the worldline AE (on the t axis) and the traveler the worldline ABDE. The journey includes a first step of constant acceleration of magnitude g starting from A, of proper time t_1, then an inertial phase of proper time t_2 then a constant deceleration step of magnitude g of proper time t_1 then an inertial phase where the rocket, landed at destination, at a distance d from the departure, will be staying for a proper time t_3, the return will be operated symmetrically. This is represented on a Cartesian diagram with coordinates t, x. In 2-dimensional Euclidean geometry, constant acceleration is represented by an arc of a circle and a constant velocity by a straight line, the slope of which is depending on the velocity, relative to initial inertial reference frame, resulting from the previous acceleration. During the first phase of constant acceleration g, the worldline is an arc of circle of length t_1, (traveler's time) in unit of time or $V.t_1$ in unit of space, of angle $\alpha = t_1.\ g$, which in radians is $\alpha V^{-1} = t_1.gV^{-1}$ and R is the radius of the circle (in units of space) $R = V^2.g^{-1}$. The angles AOB and CBD are equal to the angle a. From simple considerations on the figure, we deduce:

$$:d = 2R\left(1 - \cos(\tfrac{\alpha}{V})\right) + V.t_2\ \sin(\tfrac{\alpha}{V}) = \frac{2.V^2}{g}\left(1 - \cos(\tfrac{t_1.g}{V})\right) + v.t_2 \sin(\tfrac{t_1.g}{V})$$

From the Euclidean geometry, $L^2 = V^2t^2 + x^2$, for getting the Minkowski's one, $S^2 = -c^2T^2 + x^2$, we may, for instance, set $V^2 = -c^2$, this implying $V = i.c.$ [1],

Taking into account all these elements, one get:

$$d = \frac{2c^2}{g}(\cosh(\frac{t_1.g}{c}) - 1) + c.t_2\,.\sinh(\frac{t_1.g}{c})$$

Same type of calculation for the getting the difference of time on the 2 worldlines.

$$\Delta t = 4\left(t_1 - \frac{R}{V}\sin\frac{\alpha}{V}\right) + 2t_2\left(1 - \cos\frac{\alpha}{V}\right) \rightarrow 4\left(t_1 - \frac{c}{g}\sinh\frac{t_1.g}{c}\right) + 2t_2\left(1 - \cosh\frac{t_1.g}{c}\right)$$

The result is negative as the straight line is longer than the curve, in Minkowski's metric.

[1] Usually, we set $T = i.t$ (Wick rotation). In this case, setting $V = i.c$, which gives the same result is more convenient. Trigonometric functions become hyperbolic functions for imaginary arguments This document is adapted from: http://spoirier.lautre.net/physique.pdf.

Note: cos (i.x)=cosh (x), (1/i)sin(i.x)=sinh (x). It is straightforward to check these relations by using the polynomial definition of these functions.