INSTITUT D'ASTROPHYSIQUE DE PARIS
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Société Astronomique de France
Commission de cosmologie

THE DETECTION OF GRAVITATIONAL RADIATION

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Gravitation et Cosmologie ($\mathcal{G}\mathbb{R}\varepsilon\mathbb{C}\mathcal{O}$)
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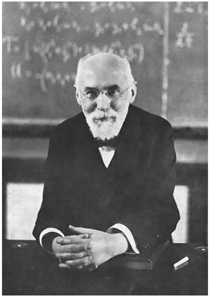
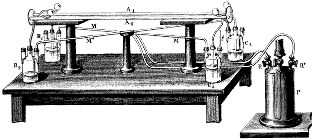
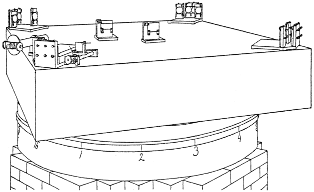
Special relativity's revolution [Lorentz 1904; Poincaré 1905; Einstein 1905]





[Fizeau 1851]



[Michelson & Morley 1887]

Einstein's equivalence principle [Einstein 1907, 1911]

1. Weak equivalence principle. All test bodies have the same acceleration in a gravitational field, independently of their mass and internal structure

$$\boxed{m_i = m_g}$$

2. Local Lorentz invariance. The result of any non gravitational experiment performed in a freely falling frame is independent of the velocity of the frame
3. Local position invariance. The result of any non gravitational experiment in a freely falling frame is independent of the position in space and time

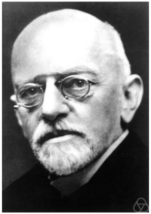
EEP is equivalent to a universal coupling of matter to the metric [Will 1993]

$$g_{\mu\nu}$$

which reduces to the Minkowski metric $\eta_{\alpha\beta}$ in freely falling frames

General Relativity: the perfect theory [Einstein, November 1915]

[Einstein & Grossmann 1912]

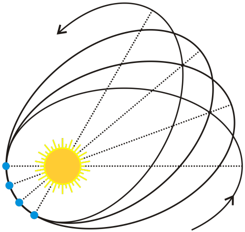


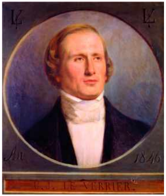
[Hilbert 1915]

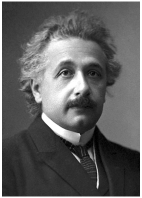
$$S_{\text{GR}} = \underbrace{\frac{c^3}{16\pi G}\int d^4x\,\sqrt{-g}\,R}_{\text{Einstein-Hilbert action}} + \underbrace{S_{\text{m}}[g_{\mu\nu}, \Psi]}_{\text{matter fields}}$$

- Field equations imply by Bianchi's identity the matter equation of motion
- General covariance implies at linear order the gauge invariance of a massless spin-2 field (the "graviton")
- System of equations is a well-posed problem ("problème bien posé") in the sense of Hadamard [Choquet-Bruhat 1952]

Mercury's precession explained [Le Verrier 1845, Einstein 1915]







1. First relativistic corrections to Newtonian gravity imply

$$\Delta\omega = \frac{6\pi G\, M_\odot}{c^2\, a(1-e^2)}\left(\frac{2+2\gamma-\beta}{3}\right) + 3\pi J_2 \left(\frac{R_\odot}{a(1-e^2)}\right)^2$$
$$= 43''/\text{century}\left[\frac{2+2\gamma-\beta}{3} + 2\,10^{-4}\left(\frac{J_2}{10^{-7}}\right)\right]$$

2. PPN parameters ($\gamma = \beta = 1$ in GR) [Eddington 1922; Nordtvedt 1968; Will 1972]
   - γ measures the spatial curvature
   - β measures the amount of non-linearity

100 years of gravitational radiation [Einstein 1916]

348 DOC. 32 INTEGRATION OF FIELD EQUATIONS

688 Sitzung der physikalisch-mathematischen Klasse vom 22. Juni 1916

Näherungsweise Integration der Feldgleichungen der Gravitation.

Von A. Einstein.

Bei der Behandlung der meisten speziellen (nicht prinzipiellen) Probleme auf dem Gebiete der Gravitationstheorie kann man sich damit begnügen, die $g_{\mu\nu}$ in erster Näherung zu berechnen. Dabei bedient man sich mit Vorteil der imaginären Zeitvariable $x_4 = it$ aus denselben Gründen wie in der speziellen Relativitätstheorie. Unter »erster Näherung« ist dabei verstanden, daß die durch die Gleichung

$$g_{\mu\nu} = -\delta_{\mu\nu} + \gamma_{\mu\nu} \qquad (1)$$

⟸ small perturbation of Minkowski's metric

definierten Größen $\gamma_{\mu\nu}$, welche linearen orthogonalen Transformationen gegenüber Tensorcharakter besitzen, gegen 1 als kleine Größen behandelt werden können, deren Quadrate und Produkte gegen die ersten Potenzen vernachlässigt werden dürfen. Dabei ist $\delta_{\mu\nu} = 1$ bzw. $\delta_{\mu\nu} = 0$, je nachdem $\mu = \nu$ oder $\mu \neq \nu$.

Wir werden zeigen, daß diese $\gamma_{\mu\nu}$ in analoger Weise berechnet werden können wie die retardierten Potentiale der Elektrodynamik.

100 years of gravitational radiation [Einstein 1918]

Einstein's quadrupole formula

mit $4\pi R^2$ multiplizierte $\bar{S}$ endlich ist der Energieverlust pro Zeiteinheit des mechanischen Systems durch Gravitationswellen. Die Rechnung ergibt

[31]

$$4\pi R^2 \bar{S} = \frac{\varkappa}{80\pi}\left[\sum_{\mu\nu} \dddot{\mathfrak{J}}_{\mu\nu}^2 - \frac{1}{3}\left(\sum_{\mu} \dddot{\mathfrak{J}}_{\mu\mu}\right)^2\right]. \qquad (30)$$

Man sieht an diesem Ergebnis, daß ein mechanisches System, welches dauernd Kugelsymmetrie behält, nicht strahlen kann, im Gegensatz zu dem durch einen Rechenfehler entstellten Ergebnis der früheren Abhandlung. [32]

Aus (27) ist ersichtlich, daß die Ausstrahlung in keiner Richtung negativ werden kann, also sicher auch nicht die totale Ausstrahlung. Bereits in der früheren Abhandlung ist betont geworden, daß das Endergebnis dieser Betrachtung, welches einen Energieverlust der Körper infolge der thermischen Agitation verlangen würde, Zweifel an der allgemeinen Gültigkeit der Theorie hervorrufen muß. Es scheint, daß eine vervollkommnete Quantentheorie eine Modifikation auch der Gravitationstheorie wird bringen müssen. [33]

§ 5 Einwirkung von Gravitationswellen auf mechanische Systeme.

Der Vollständigkeit halber wollen wir auch kurz überlegen, inwiefern Energie von Gravitationswellen auf mechanische Systeme übergehen kann. Es liege wieder ein mechanisches System vor von der

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factor 1/80 should be 1/40

§ 5 Einwirkung von Gravitationswellen auf mechanische Systeme.

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Quadrupole moment formalism [Einstein 1918; Landau & Lifchitz 1941]

1. First quadrupole formula

$$h_{ij}^{\mathsf{TT}} = \frac{2G}{c^4 D}\left\{\frac{\mathrm{d}^2 Q_{ij}}{\mathrm{d}t^2}\left(t-\frac{D}{c}\right)+\mathcal{O}\left(\frac{v}{c}\right)\right\}^{\mathsf{TT}} + \mathcal{O}\left(\frac{1}{D^2}\right)$$

2. Einstein quadrupole formula

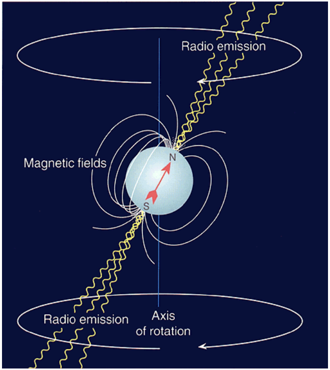
$$\left(\frac{\mathrm{d}E}{\mathrm{d}t}\right)^{\mathsf{GW}} = \frac{G}{5c^5}\left\{\frac{\mathrm{d}^3 Q_{ij}}{\mathrm{d}t^3}\frac{\mathrm{d}^3 Q_{ij}}{\mathrm{d}t^3}+\mathcal{O}\left(\frac{v}{c}\right)^2\right\}$$

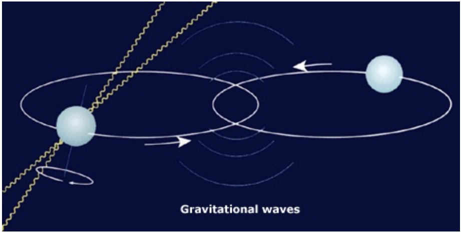
3. Radiation reaction formula [Chandrasekhar & Esposito 1970; Burke & Thorne 1970]

$$F_i^{\mathsf{reac}} = -\frac{2G}{5c^5}\rho\, x^j \frac{\mathrm{d}^5 Q_{ij}}{\mathrm{d}t^5}+\mathcal{O}\left(\frac{v}{c}\right)^7$$

which is a 2.5PN $\sim (v/c)^5$ effect in the source's equations of motion

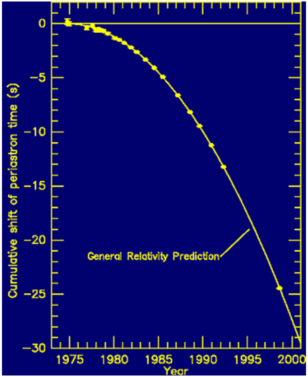
The binary pulsar PSR 1913+16 [Hulse & Taylor 1974]





- The pulsar PSR 1913+16 is a rapidly rotating neutron star emitting radio waves like a lighthouse toward the Earth
- This pulsar moves on a (quasi-)Keplerian close orbit around an unseen companion, probably another neutron star

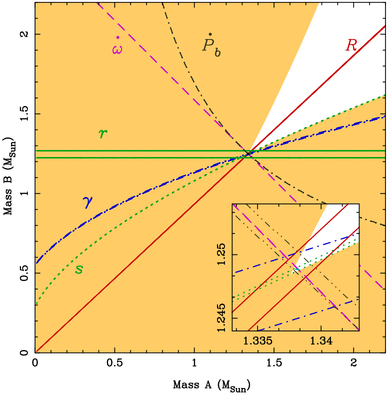
The orbital decay of the binary pulsar [Taylor & Weisberg 1982]



$$\dot{P} = -\frac{192\pi}{5c^5}\nu\left(\frac{2\pi G\,M}{P}\right)^{5/3}\frac{1+\frac{73}{24}e^2+\frac{37}{96}e^4}{(1-e^2)^{7/2}} \approx -2.4\times 10^{-12}$$

[Peters & Mathews 1963; Esposito & Harrison 1975; Wagoner 1975; Damour & Deruelle 1983]

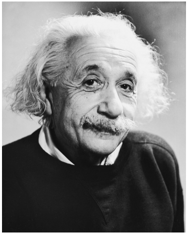
Relativistic effects in binary pulsars [e.g. Stairs 2003]



1PN order:
- $\dot{\omega}$ relativistic advance of periastron
- γ gravitational red-shift and second-order Doppler effect
- r and s range and shape of the Shapiro time delay

2.5PN order:
- $\dot{P}$ secular decrease of orbital period

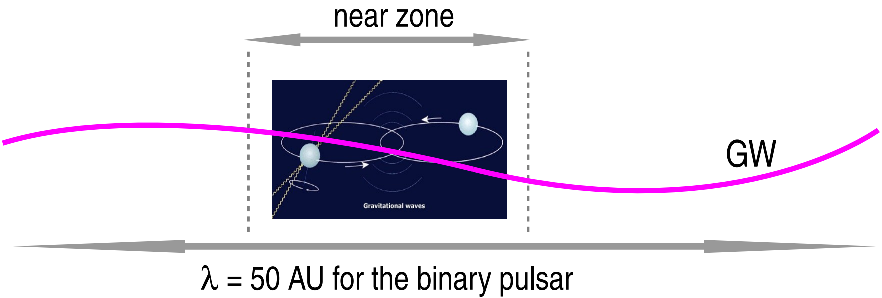
The 1PN equations of motion [Einstein, Infeld & Hoffmann 1938]



$$\frac{\mathrm{d}^2\boldsymbol{r}_A}{\mathrm{d}t^2} = -\sum_{B\neq A}\frac{Gm_B}{r_{AB}^2}\boldsymbol{n}_{AB}\Bigg[1 - 4\sum_{C\neq A}\frac{Gm_C}{c^2 r_{AC}} - \sum_{D\neq B}\frac{Gm_D}{c^2 r_{BD}}\left(1 - \frac{\boldsymbol{r}_{AB}\cdot\boldsymbol{r}_{BD}}{r_{BD}^2}\right) + \frac{1}{c^2}\left(\boldsymbol{v}_A^2 + 2\boldsymbol{v}_B^2 - 4\boldsymbol{v}_A\cdot\boldsymbol{v}_B - \frac{3}{2}(\boldsymbol{v}_B\cdot\boldsymbol{n}_{AB})^2\right)\Bigg] + \sum_{B\neq A}\frac{Gm_B}{c^2 r_{AB}^2}\boldsymbol{v}_{AB}[\boldsymbol{n}_{AB}\cdot(3\boldsymbol{v}_B - 4\boldsymbol{v}_A)] - \frac{7}{2}\sum_{B\neq A}\sum_{D\neq B}\frac{G^2 m_B m_D}{c^2 r_{AB} r_{BD}^3}\boldsymbol{n}_{BD}$$

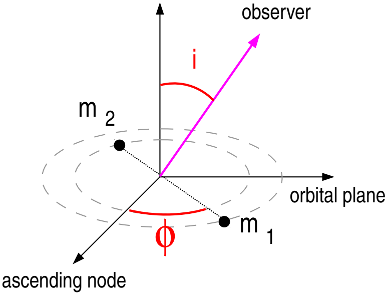
These equations were actually first obtained by [Lorentz & Droste 1917]

What is a gravitational wave?



- A gravitational wave (GW) is a ripple in the curvature of space-time propagating at the speed of light
- GWs are generated by the dynamics and orbital motion of the source
- They are more like sound waves rather than light waves

Generation of gravitational waves

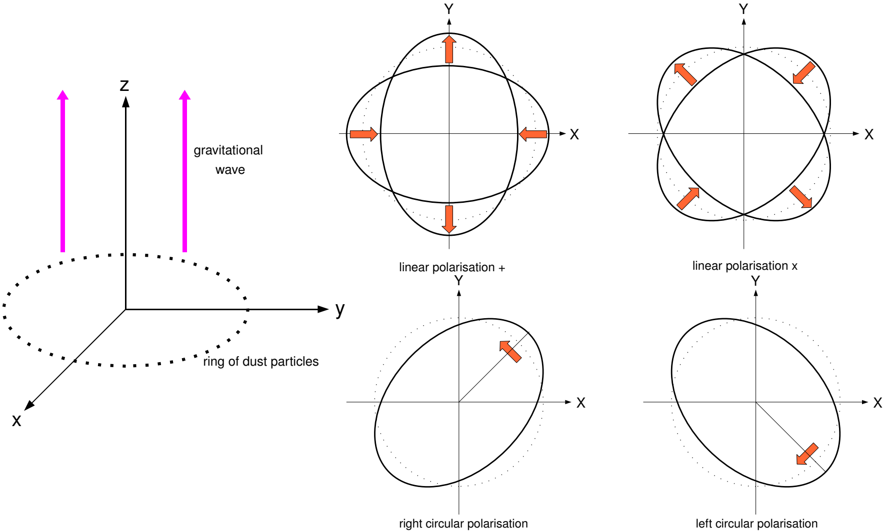


$$h_+ = \frac{2G\mu}{c^2 D}\left(\frac{GM\omega}{c^3}\right)^{2/3}\left(1+\cos^2 i\right)\cos\left(2\phi\right)$$

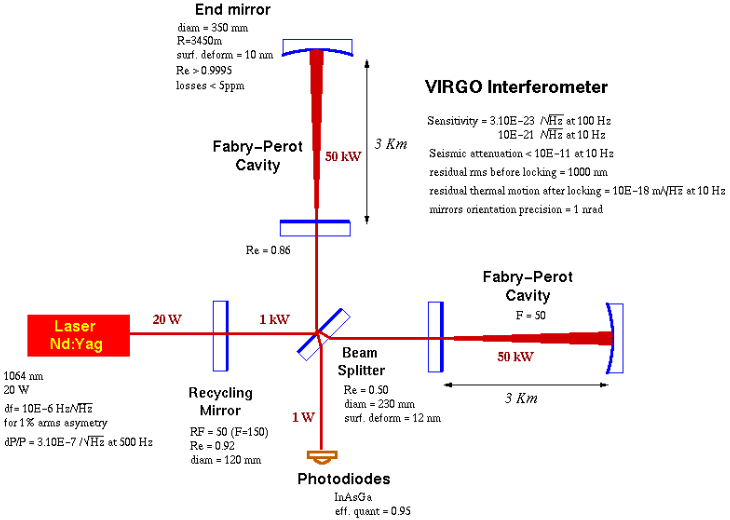
$$h_\times = \frac{2G\mu}{c^2 D}\left(\frac{GM\omega}{c^3}\right)^{2/3}\left(2\cos i\right)\sin\left(2\phi\right)$$

The distance of the source $D \equiv D_\mathrm{L}$ is measurable from the GW signal [Schutz 1986]

Polarisation modes of gravitational waves



Principle of the laser interferometric GW detector



Ground-based laser interferometric detectors

LIGO

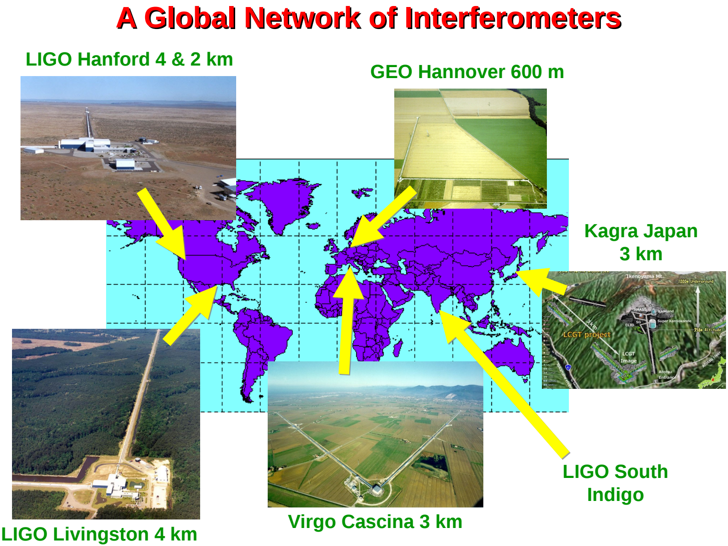
GEO

VIRGO

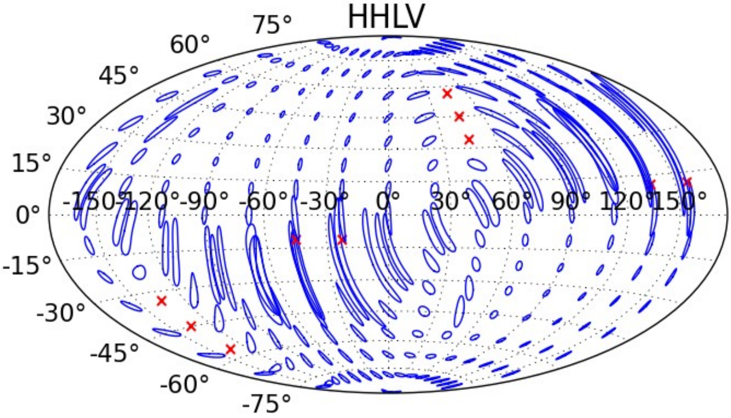
LIGO/VIRGO/GEO observe the GWs in the high-frequency band

$$10\,\mathrm{Hz} \lesssim f \lesssim 10^3\,\mathrm{Hz}$$

World-wide network of interferometric detectors

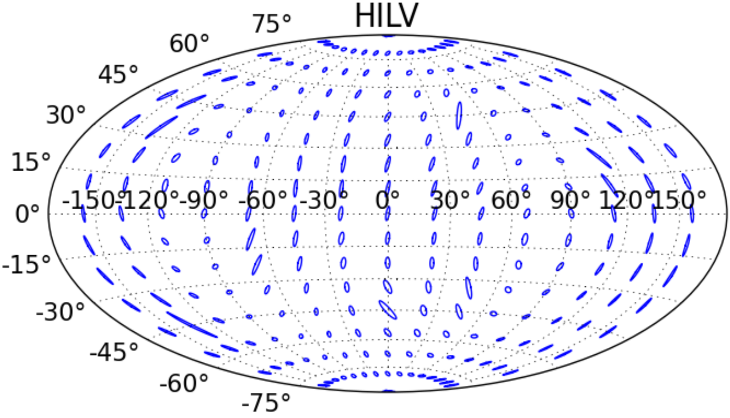


Binary neutron star merger localisation



90% localization ellipses for face-on
BNS sources @ 160 Mpc

Binary neutron star merger localisation



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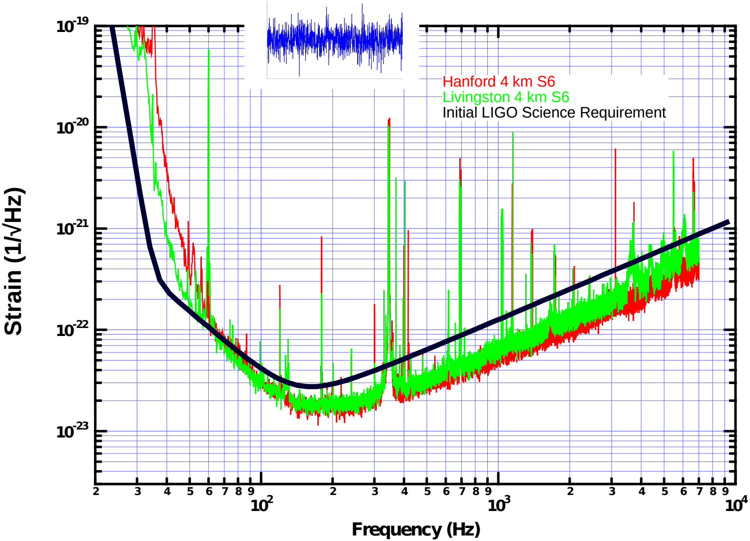
Noise curve of LIGO/VIRGO detectors



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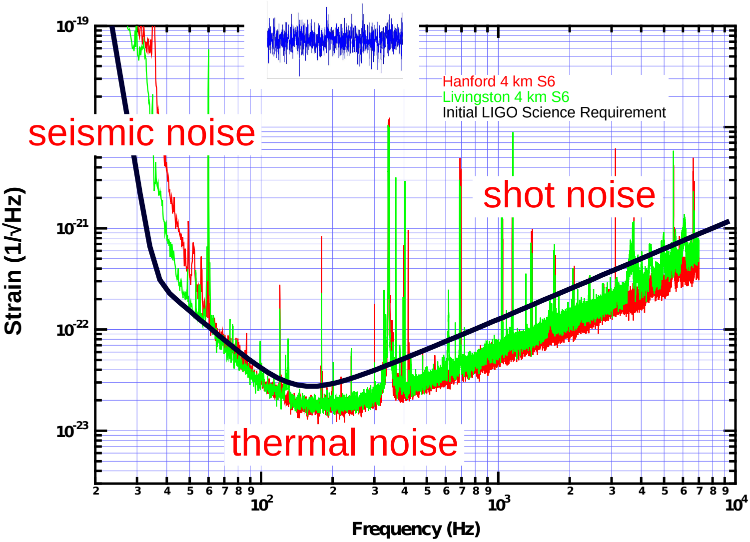
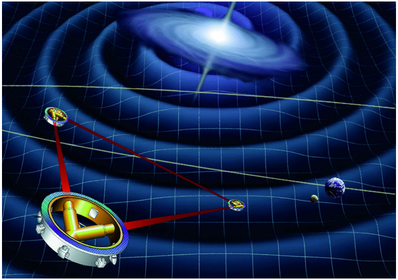
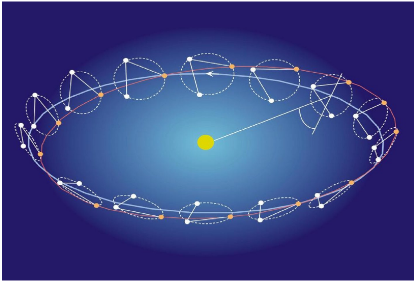


Space-based laser interferometric detector

eLISA

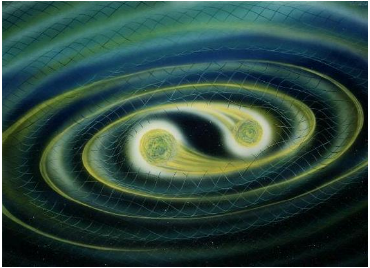




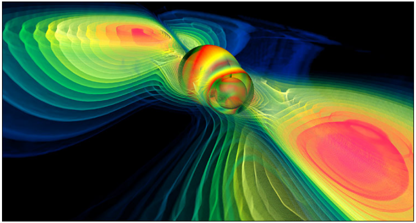
eLISA will observe the GWs in the low-frequency band

$$10^{-4}\,\mathrm{Hz} \lesssim f \lesssim 10^{-1}\,\mathrm{Hz}$$

The inspiral and merger of compact binaries



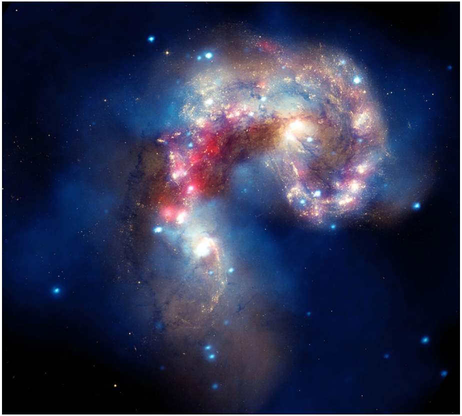
Neutron stars spiral and coalesce



Black holes spiral and coalesce

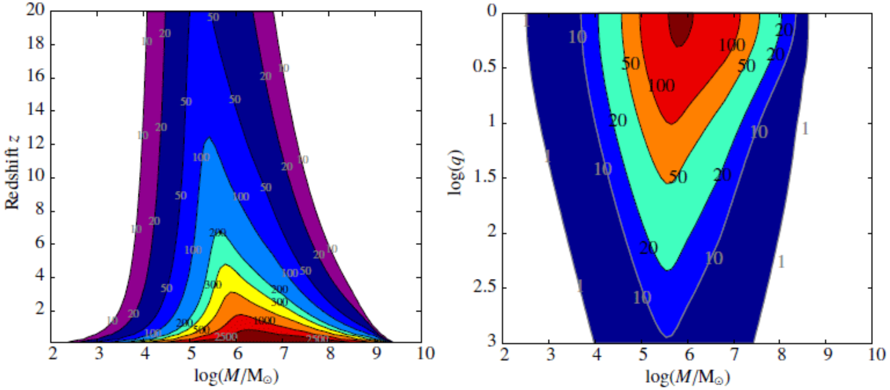
1. Neutron star ($M = 1.4\, M_\odot$) events will be detected by ground-based detectors LIGO/VIRGO/GEO/KAGRA
2. Stellar size black hole ($5\, M_\odot \lesssim M \lesssim 50\, M_\odot$) events will also be detected by ground-based detectors
3. Supermassive black hole ($10^5\, M_\odot \lesssim M \lesssim 10^8\, M_\odot$) events will be detected by the space-based detector eLISA

Coalescences of supermassive black-holes

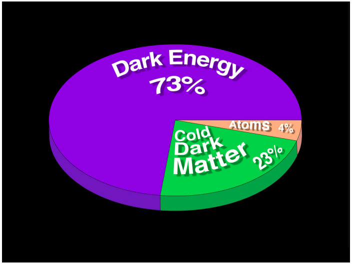


- When two galaxies collide their central supermassive black holes may form a bound binary system which will spiral and coalesce
- eLISA will be able to detect the gravitational waves emitted by such enormous events anywhere in the Universe

Supermassive black-holes detected by eLISA



Supermassive black-holes as dark energy probes

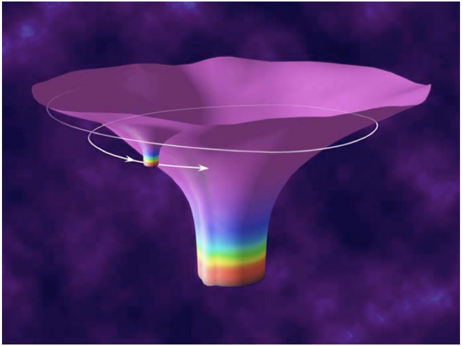


- Supermassive black-hole coalescences will be observed by eLISA up to high redshift z. In the concordance model of cosmology the distance D_L is

$$D_\mathsf{L}(z) = \frac{1+z}{H_0} \int_0^z \frac{\mathrm{d}z'}{\sqrt{\Omega_\mathsf{M}(1+z')^3 + \Omega_\mathsf{DE}(1+z')^{3(1+w)}}}$$

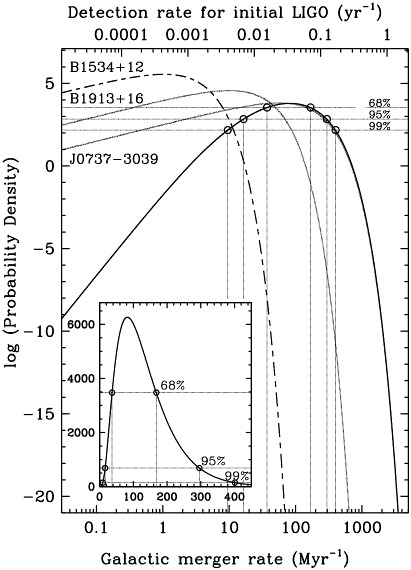
- eLISA will be able to constrain the equation of state of dark energy $w = p_\mathsf{DE}/\rho_\mathsf{DE}$ to within a few percent

Extreme mass ratio inspirals (EMRI) for eLISA



- A neutron star or a stellar black hole follows a highly relativistic orbit around a supermassive black hole. The gravitational waves generated by the orbital motion are computed using black hole perturbation theory
- Observations of EMRIs will permit to test the no-hair theorem for black holes, i.e. to verify that the central black hole is described by the Kerr geometry

Number of neutron star binaries [Kalogera *et al.* 2004]



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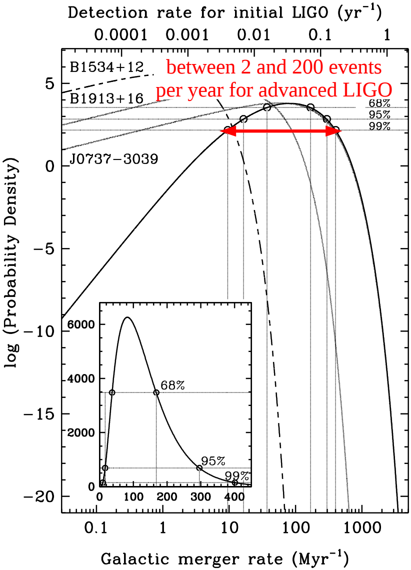
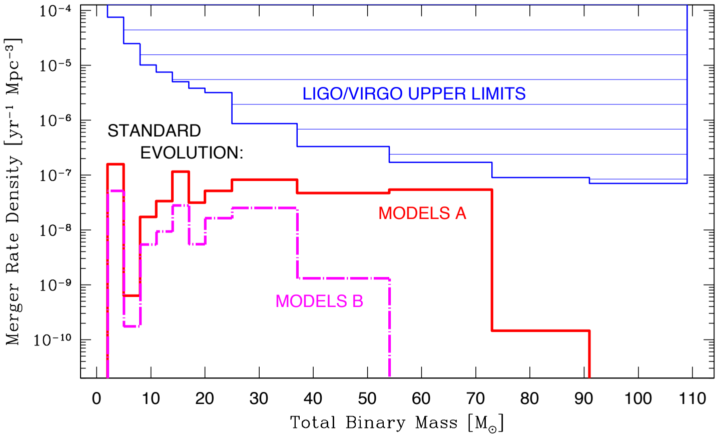


Number of black hole binaries [Belczynski *et al.* 2014]



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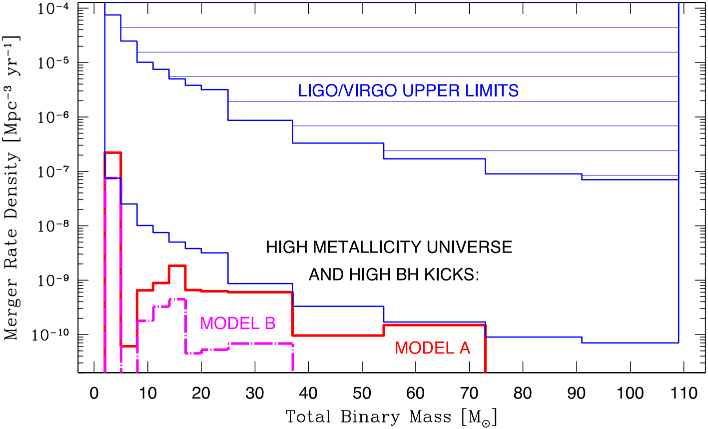
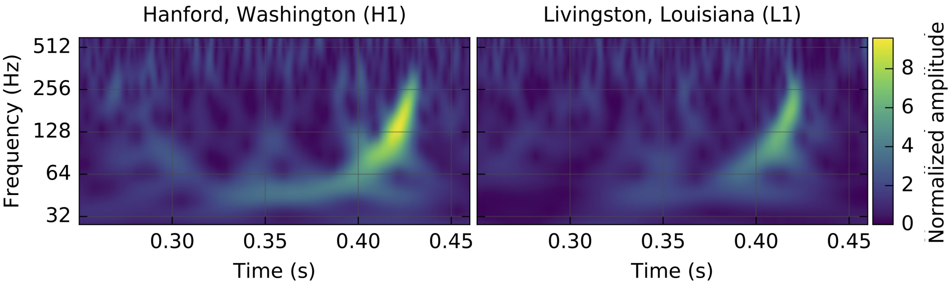


Binary black-hole event GW150914 [LIGO/VIRGO collaboration 2016]



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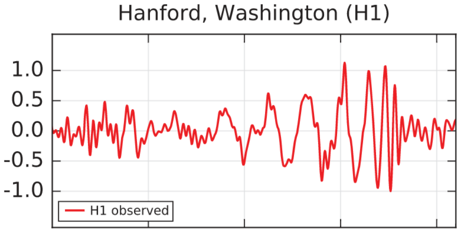
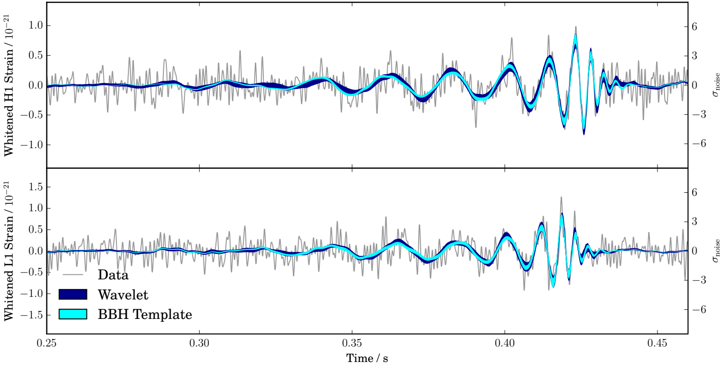
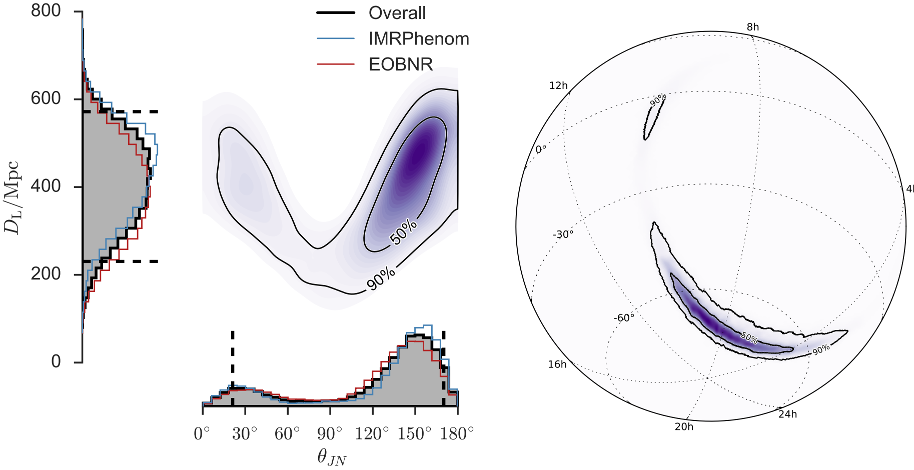
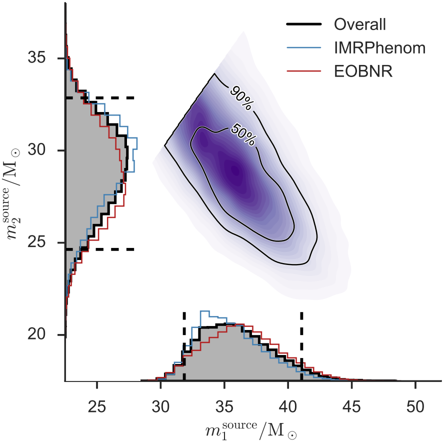


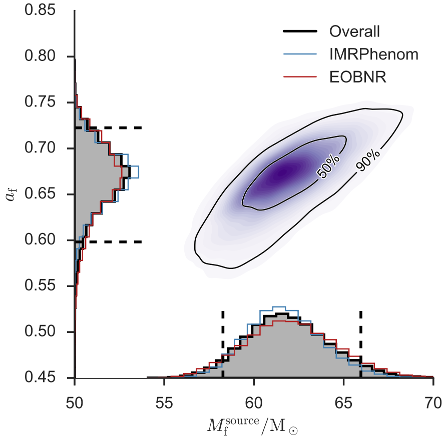


Distance & direction in the sky [LIGO/VIRGO collaboration 2016]



Measurement of masses & final spin [LIGO/VIRGO collaboration 2016]





Power & gravitational wave amplitude

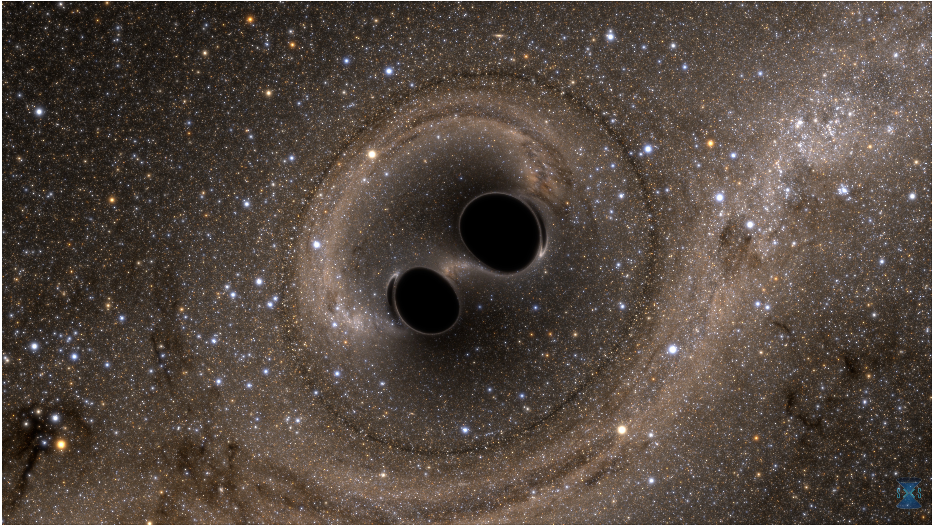
- Power released in gravitational waves

$$\boxed{\mathcal{P}^{\mathsf{GW}} \sim \frac{3M_\odot c^2}{0.2\,\mathsf{s}} \sim 10^{55}\,\mathsf{erg/s} \sim 10^{-4}\,\frac{c^5}{G}}$$

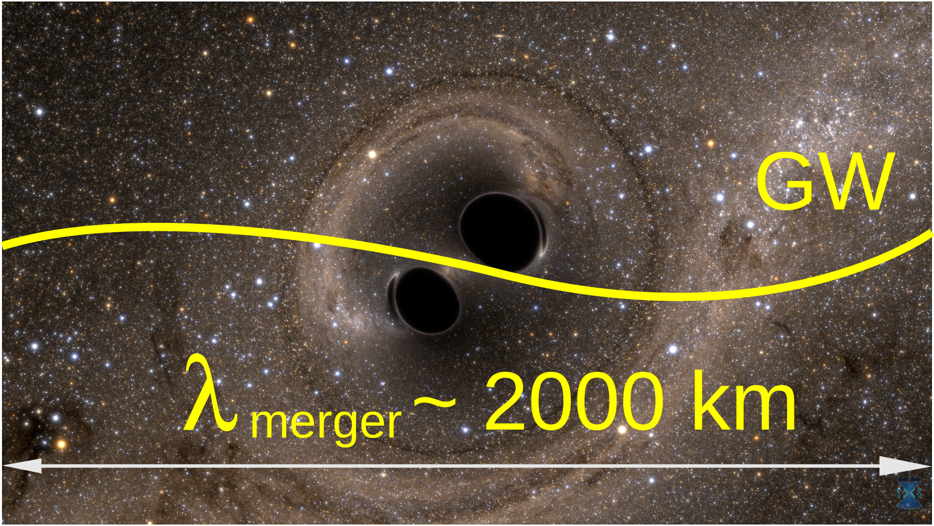
- Amplitude of the gravitational wave

$$\begin{aligned} h_{\mathsf{eff}} &\sim h\sqrt{\frac{\omega^2}{\dot{\omega}}} \\ &\sim 4.1 \times 10^{-22} \left(\frac{\mu}{M_\odot}\right)^{1/2} \left(\frac{M}{M_\odot}\right)^{1/3} \left(\frac{100\,\mathsf{Mpc}}{D}\right) \left(\frac{100\,\mathsf{Hz}}{f_{\mathsf{merger}}}\right)^{-1/6} \\ &\sim 1.6 \times 10^{-21} \end{aligned}$$

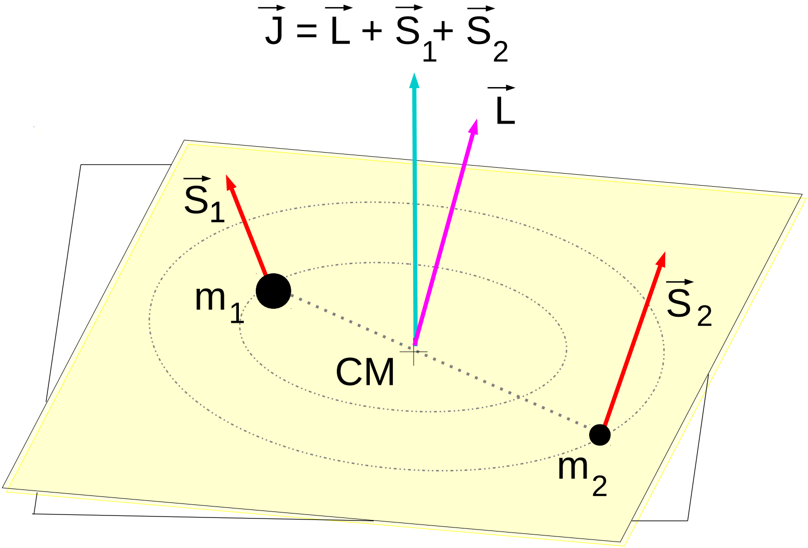
Artist view of GW150914



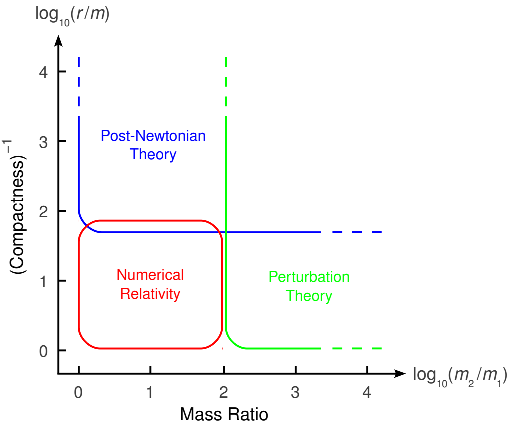
Artist view of GW150914



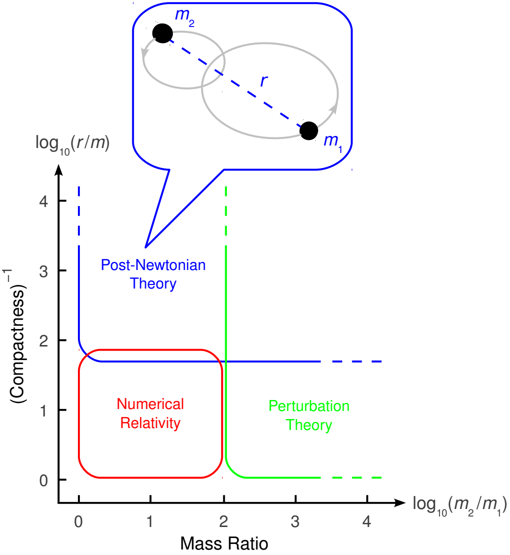
Modelling the compact binary inspiral



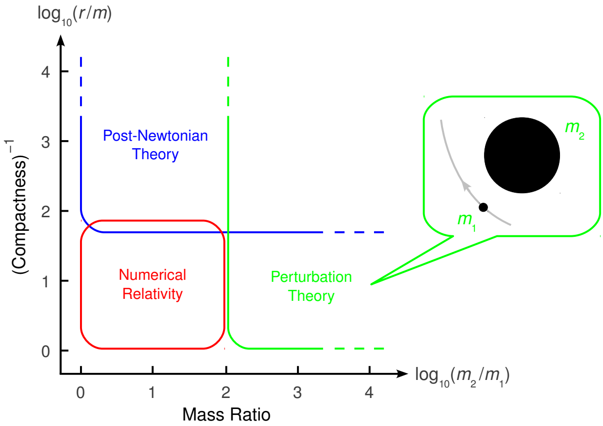
Methods to compute GW templates



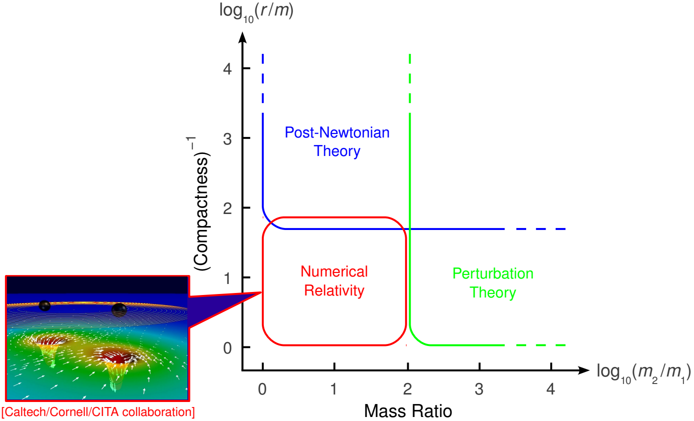
Methods to compute GW templates



Methods to compute GW templates

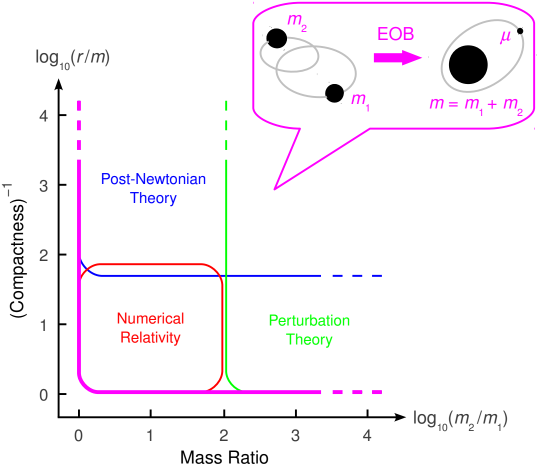


Methods to compute GW templates



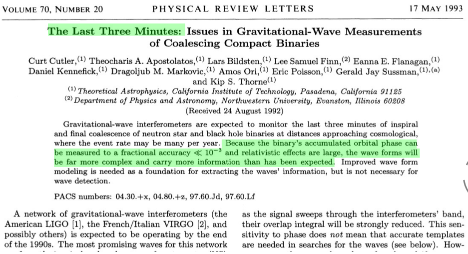
Methods to compute GW templates

[Buonanno & Damour 1998]



Inspiralling binaries require high-order PN modelling

[Caltech "3mn paper" 1992; Blanchet & Schäfer 1993]



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The Last Three Minutes: Issues in Gravitational-Wave Measurements of Coalescing Compact Binaries

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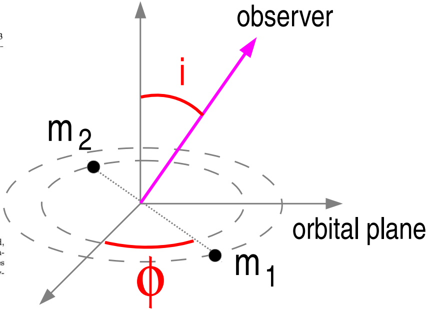
(Received 24 August 1992)

Gravitational-wave interferometers are expected to monitor the last three minutes of inspiral and final coalescence of neutron star and black hole binaries at distances approaching cosmological, where the event rate may be many per year. Because the binary's accumulated orbital phase can be measured to a fractional accuracy $\ll 10^{-3}$ and relativistic effects are large, the wave forms will be far more complex and carry more information than has been expected. Improved wave form modeling is needed as a foundation for extracting the waves' information, but is not necessary for wave detection.

PACS numbers: 04.30.+x, 04.80.+z, 97.60.Jd, 97.60.Lf

A network of gravitational-wave interferometers (the American LIGO [1], the French/Italian VIRGO [2], and possibly others) is expected to be operating by the end of the 1990s. The most promising waves for this network ...

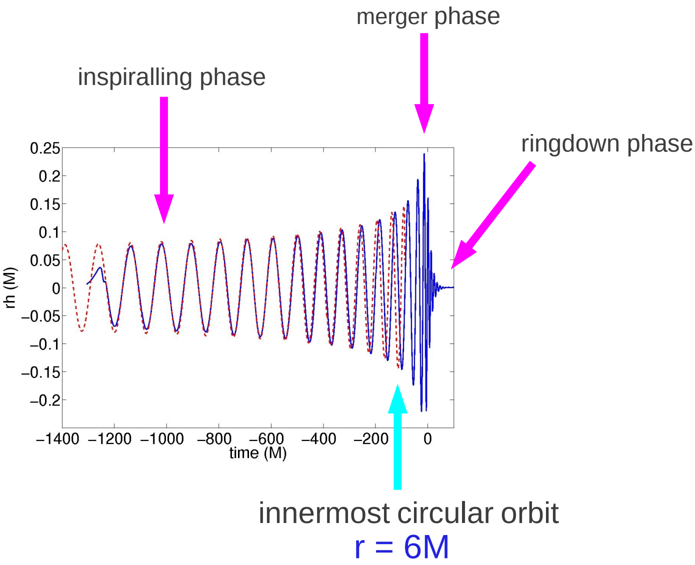
... as the signal sweeps through the interferometers' band, their overlap integral will be strongly reduced. This sensitivity to phase does *not* mean that accurate templates are needed in searches for the waves (see below). How-



$$\phi(t) = \phi_0 - \underbrace{\frac{M}{\mu}\left(\frac{GM\omega}{c^3}\right)^{-5/3}}_{\text{quadrupole formalism}} \Bigg\{ 1 + \underbrace{\frac{1\text{PN}}{c^2} + \frac{1.5\text{PN}}{c^3} + \cdots + \frac{3\text{PN}}{c^6} + \cdots}_{\text{needs to be computed with 3PN precision at least}} \Bigg\}$$

Here 3PN means 5.5PN as a radiation reaction effect !

The gravitational chirp of compact binaries



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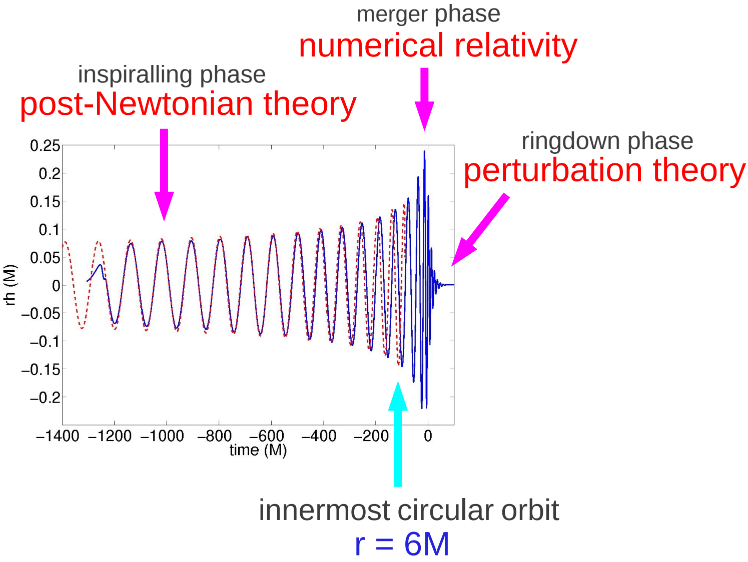

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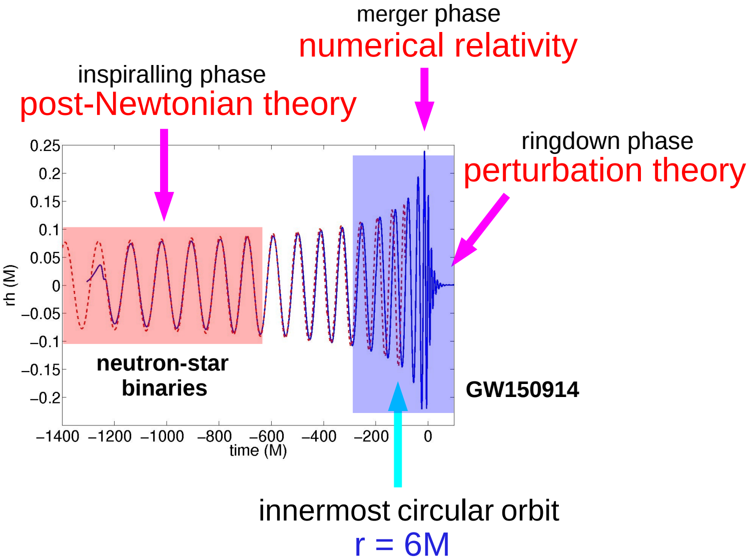
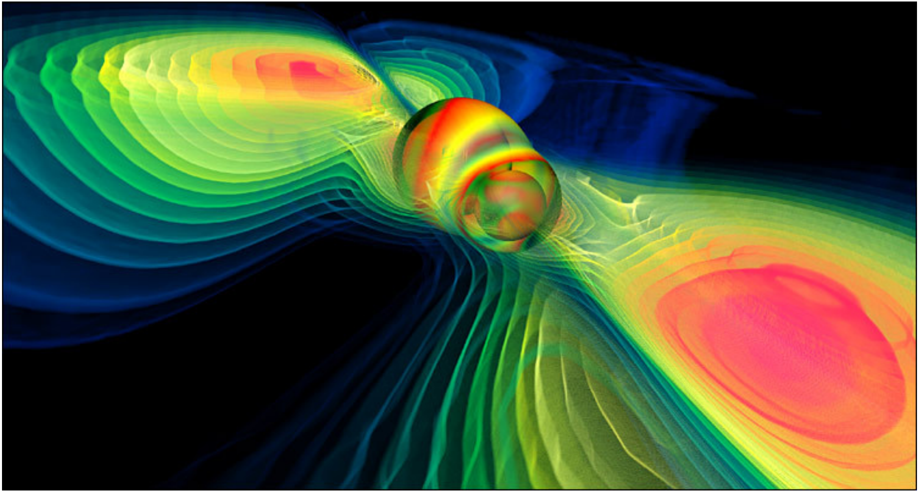
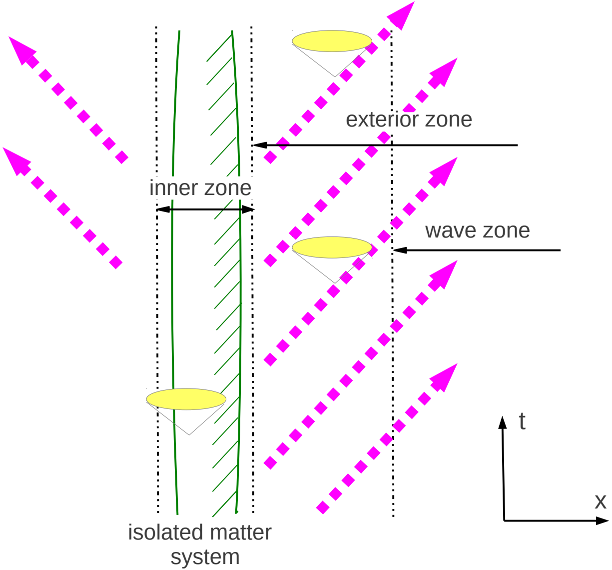


Breakthrough of numerical relativity

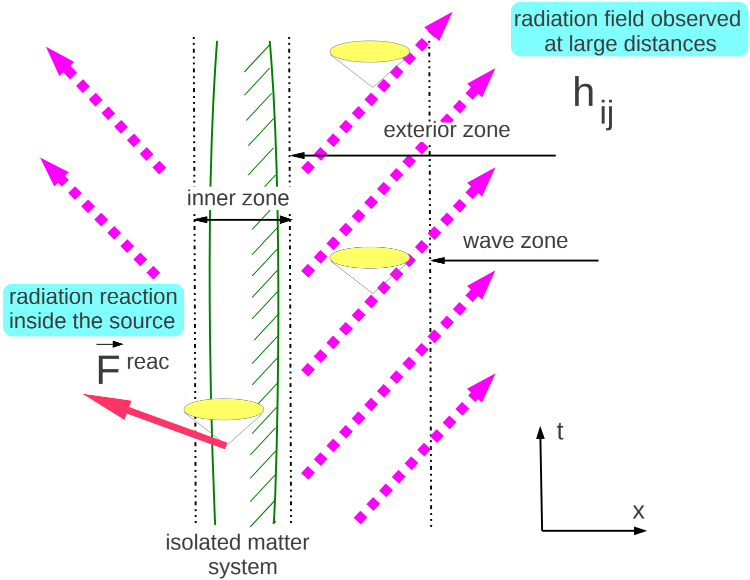
[Pretorius 2005; Baker *et al.* 2006; Campanelli *et al.* 2006]



Gravitational radiation from isolated systems

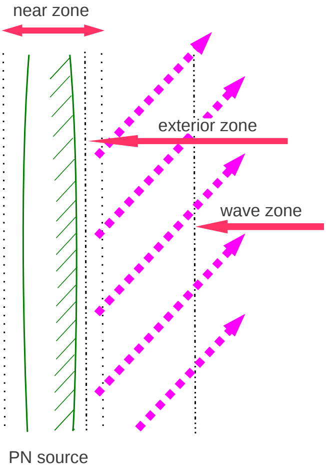


Gravitational radiation from isolated systems



The MPM-PN formalism [Blanchet & Damour 1986, 1988; Blanchet 1987, 1993]

A multipolar post-Minkowskian (MPM) expansion in the exterior zone is matched to a general post-Newtonian (PN) expansion in the near zone



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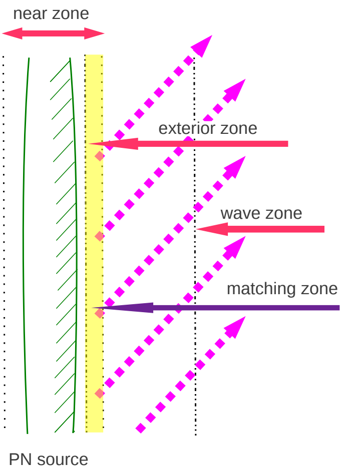

4PN equations of motion of compact binaries

$$\frac{\mathrm{d}v_1^i}{\mathrm{d}t} = -\frac{Gm_2}{r_{12}^2}n_{12}^i + \overbrace{\frac{1}{c^2}\left\{\left[\frac{5G^2m_1m_2}{r_{12}^3}+\frac{4G^2m_2^2}{r_{12}^3}+\cdots\right]n_{12}^i+\cdots\right\}}^{\text{1PN Lorentz-Droste-Einstein-Infeld-Hoffmann term}}$$
$$+\underbrace{\frac{1}{c^4}[\cdots]}_{\text{2PN}}+\underbrace{\frac{1}{c^5}[\cdots]}_{\substack{\text{2.5PN}\\\text{radiation reaction}}}+\underbrace{\frac{1}{c^6}[\cdots]}_{\text{3PN}}+\underbrace{\frac{1}{c^7}[\cdots]}_{\substack{\text{3.5PN}\\\text{radiation reaction}}}+\underbrace{\frac{1}{c^8}[\cdots]}_{\substack{\text{4PN}\\\text{conservative \& radiation tail}}}+\mathcal{O}\left(\frac{1}{c^9}\right)$$

2PN:

[Otha, Okamura, Kimura & Hiida 1973, 1974; Damour & Schäfer 1985]	ADM Hamiltonian
[Damour & Deruelle 1981; Damour 1983]	Harmonic coordinates
[Kopeikin 1985; Grishchuk & Kopeikin 1986]	Extended fluid balls
[Blanchet, Faye & Ponsot 1998]	Direct PN iteration
[Itoh, Futamase & Asada 2001]	Surface integral method

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3PN	[Jaranowski & Schäfer 1999; Damour, Jaranowski & Schäfer 2001]	ADM Hamiltonian
	[Blanchet & Faye 2000; de Andrade, Blanchet & Faye 2001]	Harmonic equations of motion
	[Itoh & Futamase 2003; Itoh 2004]	Surface integral method
	[Foffa & Sturani 2011]	Effective field theory
4PN	[Jaranowski & Schäfer 2013; Damour, Jaranowski & Schäfer 2014]	ADM Hamiltonian
	[Bernard, Blanchet, Bohé, Faye & Marsat 2015]	Fokker Lagrangian

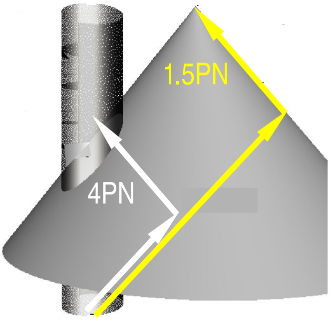
3.5PN energy flux of compact binaries

[BDIWW 1995; B 1996, 1998; BFIJ 2002; BDEI 2006]

$$\mathcal{F} = \frac{32c^5}{5G}\nu^2 x^5 \Bigg\{ 1 + \overbrace{\left(-\frac{1247}{336} - \frac{35}{12}\nu\right) x}^{\text{1PN}} + \overbrace{4\pi x^{3/2}}^{\text{1.5PN tail}}$$
$$+ \left(-\frac{44711}{9072} + \frac{9271}{504}\nu + \frac{65}{18}\nu^2\right) x^2 + \overbrace{\left(-\frac{8191}{672} - \frac{583}{24}\nu\right)\pi x^{5/2}}^{\text{2.5PN tail}}$$
$$+ \Bigg[\frac{6643739519}{69854400} + \overbrace{\frac{16}{3}\pi^2 - \frac{1712}{105}\gamma_\mathsf{E} - \frac{856}{105}\ln(16\,x)}^{\text{3PN tail-of-tail}}$$
$$+ \left(-\frac{134543}{7776} + \frac{41}{48}\pi^2\right)\nu - \frac{94403}{3024}\nu^2 - \frac{775}{324}\nu^3 \Bigg] x^3$$
$$+ \underbrace{\left(-\frac{16285}{504} + \frac{214745}{1728}\nu + \frac{193385}{3024}\nu^2\right)\pi x^{7/2}}_{\text{3.5PN tail}} + \mathcal{O}\left(\frac{1}{c^8}\right) \Bigg\}$$

Tails of gravitational waves [*e.g.* Bonnor 1959]

Tails are produced by backscatter of GWs on the curvature induced by the matter source's total mass M



$$\delta h_{ij}^{\text{tail}} = \frac{4G}{c^4 r} \underbrace{\frac{GM}{c^3} \int_{-\infty}^{t} \mathrm{d}t' Q_{ij}(t') \ln\left(\frac{t-t'}{\tau_0}\right)}_{\text{The tail is dominantly a 1.5PN effect}} + \cdots$$

Bounds on PN parameters [LIGO/VIRGO collaboration 2016]